CHAPTER 5

ARITHMETIC PROGRESSION

ONE MARK QUESTIONS

Multiple Choice Questions

1. The n^{th} term of the AP $a, 3a, 5a, ...$ is

(a) na (b) $(2n-1)a$

(c) $(2n+1)a$ (d) $2na$

Ans : [Board 2020 OD Standard]

Given AP is $a, 3a, 5a, ...$

First term is a and $d = 3a - a = 2a$

n^{th} term
$$a_n = a + (n-1)d$$
$$= a + (n-1)2a$$
$$= a + 2na - 2a$$
$$= 2na - a = (2n-1)a$$

Thus (b) is correct option.

2. The common difference of the AP $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}, ...$ is

(a) 1 (b) $\frac{1}{p}$

(c) -1 (d) $-\frac{1}{p}$

Ans : [Board 2020 OD Standard]

Given AP is $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p} ...$

Common difference

$$d = \frac{1-p}{p} - \frac{1}{p} = \frac{1-p-1}{p} = \frac{-p}{p} = -1$$

Thus (c) is correct option.

3. The value of x for which $2x, (x+10)$ and $(3x+2)$ are the three consecutive terms of an AP, is

(a) 6 (b) -6

(c) 18 (d) -18

Ans : [Board 2020 Delhi Standard]

Since $2x$, $(x+10)$ and $(3x+2)$ are in AP we obtain,

$$(x+10) - 2x = (3x+2) - (x+10)$$
$$-x + 10 = 2x - 8$$
$$-x - 2x = -8 - 10$$
$$-3x = -18 \Rightarrow x = 6$$

Thus (a) is correct option.

4. The first term of AP is p and the common difference is q, then its 10th term is

(a) $q+9p$ (b) $p-9q$

(c) $p+9q$ (d) $2p+9q$

Ans : [Board 2020 Delhi Standard]

We have $a = p$ and $d = q$

$$a_{10} = a + (10-1)$$
$$= p + 9q$$

Thus (c) is correct option.

5. In an AP, if $d = -4$, $n = 7$ and $a_n = 4$, then a is equal to

(a) 6 (b) 7

(c) 20 (d) 28

Ans : (d) 28

In an AP,
$$a_n = a + (n-1)d$$
$$4 = a + (7-1)(-4)$$
$$4 = a + 6(-4)$$
$$4 + 24 = a \Rightarrow a = 28$$

Thus (d) is correct option.

6. In an AP, if $a = 3.5$, $d = 0$ and $n = 101$, then a_n will be

(a) 0 (b) 3.5

(c) 103.5 (d) 104.5

Ans : (b) 3.5

As, $d = 0$ all the terms are same whatever the value of n. So, $a_n = 3.5$.

Alternate Method :

In an AP, $a_n = a + (n-1)d$

$$a_n = 3.5 + (101-1) \times 0 = 3.5$$

Thus (b) is correct option.

7. The 11th term of an AP $-5, \frac{-5}{2}, 0, \frac{5}{2}, ...,$ is

(a) -20 (b) 20
(c) -30 (d) 30

Ans : (b) 20

Here, $a = -5,\ d = \frac{-5}{2} - (-5) = \frac{5}{2}$

nth term, $a_n = a + (n-1)d$

$$a_{11} = -5 + (11-1) \times \left(\frac{5}{2}\right)$$

$$a_{11} = -5 + 25 = 20$$

Thus (b) is correct option.

8. In an AP, if $a = 3.5$, $d = 0$ and $n = 101$, then a_n will be

(a) 0 (b) 3.5
(c) 103.5 (d) 104.5

Ans : (b) 3.5

For an AP, $a_n = a + (n-1)d$

$$= 3.5 + (101-1) \times 0$$

$$= 3.5$$

Thus (b) is correct option.

9. Which term of an AP, 21, 42, 63, 84, ... is 210?

(a) 9th (b) 10th
(c) 11th (d) 12th

Ans : (b) 10th

Let nth term of given AP be 210,

First term, $a = 21$

Common difference, $d = 42 - 21 = 21$

and $a_n = 210$

In an AP, $a_n = a + (n-1)d$

$$210 = 21 + (n-1)21$$

$$210 = 21 + 21n - 21$$

$$210 = 21n \Rightarrow n = 10$$

Hence, the 10th term of the given AP is 210.

Thus (b) is correct option.

10. If the common difference of an AP is 5, then what is $a_{18} - a_{13}$?

(a) 5 (b) 20
(c) 25 (d) 30

Ans : (c) 25

Given, the common difference of AP i.e, $d = 5$

Using, $a_n = a + (n-1)d$

We have, $a_{18} = a + (18-1)d$

and $a_{13} = a + (13-1)d$

Now, $a_{18} - a_{13} = a + (18-1)d - [a + (13-1)d]$

$$= a + 17 \times 5 - a - 12 \times 5$$

$$= 85 - 60 = 25$$

Thus (c) is correct option.

11. What is the common difference of an AP in which $a_{18} - a_{14} = 32$?

(a) 8 (b) -8
(c) -4 (d) 4

Ans : (a) 8

We have $a_{18} - a_{14} = 32$

In an AP, $a_n = a + (n-1)d$

$$a + (18-1)d - [a + (14-1)d] = 32$$

$$a + 17d - a - 13d = 32$$

$$4d = 32 \Rightarrow d = 8$$

Hence, the required common difference of the given AP is 8.

Thus (a) is correct option.

12. The 4th term from the end of an AP $-11, -8, -5, ..., 49$ is

(a) 37 (b) 40
(c) 43 (d) 58

Ans : (b) 40

Common difference,

$$d = -8 - (-11) = -8 + 11 = 3$$

Last term, $l = 49$

nth term of an AP from the end is

$$a_n = l - (n-1)d$$

, $a_4 = 49 - (4-1) \times 3$

$$= 49 - 9 = 40$$

13. If the first term of an AP is -5 and the common difference is 2, then the sum of the first 6 terms is

(a) 0 (b) 5

(c) 6 (d) 15

Ans : (a) 0

We have $a = -5$ and $d = 2$

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$S_6 = \frac{6}{2}[2a + (6-1)d]$$

$$= 3[2(-5) + 5(2)]$$

$$= 3(-10 + 10) = 0$$

Thus (a) is correct option.

14. The sum of first 16 terms of the AP 10, 6, 2, ... is

(a) -320 (b) 320

(c) -352 (d) -400

Ans : (a) -320

Given, AP, is 10, 6, 2 .....

We have $a = 10$ and $d = (6 - 10) = -4$

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$S_{16} = \frac{16}{2}[2a + (16-1)d]$$

$$= 8[2 \times 10 + 15(-4)]$$

$$= 8(20 - 60)$$

$$= 8(-40) = -320$$

Thus (a) is correct option.

15. In an AP, if $a = 1$, $a_n = 20$ and $S_n = 399$, then n is equal to

(a) 19 (b) 21

(c) 38 (d) 42

Ans : (c) 38

We have $a = 1,\ a_n = 20$ and $S_n = 399$

Now, $S_n = \frac{n}{2}(a + a_n)$

$$399 = \frac{n}{2}(1 + 20)$$

$$n = \frac{399 \times 2}{21} = 38.$$

16. The sum of first five multiples of 3 is

(a) 45 (b) 55

(c) 65 (d) 75

Ans : (a) 45

The first five multiples of 3 are 3, 6, 9, 12 and 15.
Here, first term, $a = 3$, $d = 6 - 3 = 3$ and $n = 5$

$$S_n = \frac{n}{2}\{2a + (n-1)\}d$$

$$S_5 = \frac{5}{2}[2a + (5-1)d]$$

$$= \frac{5}{2}[2 \times 3 + 4 \times 3]$$

$$= \frac{5}{2}(6 + 12) = \frac{5}{2} \times 18 = 45$$

Thus (a) is correct option.

17. If the sum of the series $2 + 5 + 8 + 11$ .... is 60100, then the number of terms are

(a) 100 (b) 200

(c) 150 (d) 250

Ans : (b) 200

We have $a = 2.\ d = 5 - 2 = 3$ and $S_n = 60100$

$$\frac{n}{2}[2a + (n-1)d] = S_n$$

$$\frac{n}{2}[4 + (n-1)3] = 60100$$

$$n(3n + 1) = 120200$$

$$3n^2 + n - 120200 = 0$$

$$(n - 200)(3n + 601) = 0 \Rightarrow n = 200, \frac{601}{3}$$

Thus $n = 200$ because n can not be fraction.

Thus (b) is correct option.

18. If the common difference of an AP is 5, then what is $a_{18} - a_{13}$?

(a) 5 (b) 20

(c) 25 (d) 30

Ans : (c) 25

Given, the common difference of AP i.e., $d = 5$

Now $a_n = a + (n-1)d$

Now, $a_{18} - a_{13} = a + (18-1)d - [a + (13-1)d]$

$= a + 17 \times 5 - a - 12 \times 5$

$= 85 - 60 = 25$

Thus (c) is correct option.

19. There are 60 terms is an AP of which the first term is 8 and the last term is 185. The 31^{st} term is

(a) 56 (b) 94

(c) 85 (d) 98

Ans : (d) 98

Let d be the common difference;

Now $a_n = a + (n-1)d$

Then 60^{th} term, $a_{60} = 8 + (60-1)d$

$185 = 8 + 59d$

$59d = 177 \Rightarrow d = 3$

31^{th} term $a_{31} = 8 + 30 \times 3 = 98$

Thus (d) is correct option.

20. The first and last term of an AP are a and ℓ respectively. If S is the sum of all the terms of the AP and the common difference is $\dfrac{\ell^2 - a^2}{k - (\ell + a)}$, then k is equal to

(a) S (b) $2S$

(c) $3S$ (d) None of these

Ans : (b) $2S$

We have, $S = \frac{n}{2}(a + \ell)$

$$\frac{2S}{a+\ell} = n \qquad (1)$$

Also, $\ell = a + (n-1)d$

$$d = \frac{\ell - a}{n-1} = \frac{\ell - a}{\frac{2S}{a+\ell} - 1}$$

$$= \frac{\ell^2 - a^2}{2S - (\ell + a)}$$

Thus $k = 2S$

Thus (b) is correct option.

21. If the nth term of an AP is given by $a_n = 5n - 3$, then the sum of first 10 terms if

(a) 225 (b) 245

(c) 255 (d) 270

Ans : (b) 245

We have $a_n = 5n - 3$

Substituting $n = 1$ and 10 we have

$a = 2$

$a_{10} = 47$

Thus $S_n = \frac{n}{2}(a + a_n)$

$S_{10} = \frac{10}{2}(2 + 47)$

$= 5 \times 49 = 245$

Thus (b) is correct option.

22. Two APs have the same common difference. The first term of one of these is -1 and that of the other is -8. Then the difference between their 4th terms is

(a) -1 (b) -8

(c) 7 (d) -9

Ans : (c) 7

4th term of first AP,

$a_4 = -1 + (4-1)d = -1 + 3d$

and 4th term of second AP,

$a'_4 = -8 + (4-1)d = -8 + 3d$

Now, the difference between their 4th terms,

$a'_4 - a_4 = (-1 + 3d) - (-8 + 3d)$

$= -1 + 3d + 8 - 3d = 7$

Hence, the required difference is 7.

Thus (c) is correct option.

23. An AP starts with a positive fraction and every alternate term is an integer. If the sum of the first 11 terms is 33, then the fourth term is

(a) 2 (b) 3

(c) 5 (d) 6

Ans : (a) 2

We have $S_{11} = 33$

$$\frac{11}{2}[2a+10d] = 33$$

$$a+5d = 3$$

i.e. $a_6 = 3 \Rightarrow a_4 = 2$

Since, alternate terms are integers and the given sum is possible, $a_4 = 2$.

Thus (a) is correct option.

24. If the sum of the first $2n$ terms of 2, 5, 8, ... is equal to the sum of the first n terms of 57, 59, 61, ..., then n is equal to

(a) 10 (b) 12

(c) 11 (d) 13

Ans : (c) 11

$$\frac{2n}{2}\{2\times2+(2n-1)3\} = \frac{n}{2}\{2\times57+(n-1)2\}$$

$$2(6n+1) = 112+2n$$

$$10n = 110 \Rightarrow n = 11$$

Thus (c) is correct option.

25. In an AP, if $d=-4$, $n=7$ and $a_n=4$, then a is equal to

(a) 6 (b) 7

(c) 20 (d) 28

Ans : (d) 28

In an AP, $a_n = a+(n-1)d$

$$4 = a+(7-1)(-4)$$

$$4 = a+6(-4)$$

$$4+24 = a \Rightarrow a = 28$$

Thus (d) is correct option.

26. The first four terms of an AP whose first term is -2 and the common difference is -2 are

(a) $-2,0,2,4$ (b) $-2,4,-8,16$

(c) $-2,-4,-6,-8$ (d) $-2,-4,-8,-16$

Ans : (c) $-2,-4,-6,-8$

Let the first four terms of an AP are $a, a+d, a+2d$ and $a+3d$.

Given, that first term, $a=-2$ and common difference, $d=-2$, then we have an AP as follows

$$-2,\quad -2-2,\quad -2+2(-2),\quad -2+3(-2)$$
$$=-2,-4,-6,-8$$

Thus (c) is correct option.

27. The 21[th] term of an AP whose first two terms are -3 and 4, is

(a) 17 (b) 137

(c) 143 (d) -143

Ans : (b) 137

Given, first two terms of an AP are

$$a =-3$$

and $a+d = 4$

$$-3+d = 4 \Rightarrow d = 7$$

For an AP, $a_n = a+(n-1)d$

Thus $a_{21} = a+(21-1)d$

$$=-3+(20)7$$
$$=-3+140 = 137$$

Thus (b) is correct option.

28. The number of two digit numbers which are divisible by 3 is

(a) 33 (b) 31

(c) 30 (d) 29

Ans : (c) 30

Two digit numbers which are divisible by 3 are 12, 15, 18, ..., 99;

Here $a=12$, $d=3$ and $a_n=99$

For an AP, $a_n = a+(n-1)d$

So, $99 = 12+(n-1)\times3$

$$99-12 = 3n-3$$
$$99-12+3 = 3n$$
$$90 = 3n \Rightarrow n = 30$$

Thus (c) is correct option.

29. The list of numbers $-10, -6, -2, 2,$ ..... is

(a) an AP with $d=-16$ (b) an AP with $d=4$

(c) an AP with $d = -4$ (d) not an AP

Ans : (b) an AP with $d = 4$

The given numbers are $-10, -6, -2, 2, ...$

Here, $a_1 = 10, a_2 = -6, a_3 = -2$ and $a_4 = 2, ...$

Since, $d_1 = a_2 - a_1 = -6 - (-10) = -6 + 10 = 4$

$d_2 = a_3 - a_2 = -2 - (-6) = -2 + 6 = 4$

$d_3 = a_4 - a_3 = 2 - (-2) = 2 + 2 = 4$

Since, $d_1 = d_2 = d_3 = ... = 4$

i.e., each successive term of given list has same difference. So, the given list forms an AP with common difference, $d = 4$.

Thus (b) is correct option.

30. If the nth term of an AP is $4n + 1$, then the common difference is

(a) 3 (b) 4

(c) 5 (d) 6

Ans : (b) 4

Given that the n^{th} term of an AP is $4n + 1$.

$$a_n = 4n + 1$$

Substituting $n = 1, 2, 3, ...$ we have

$$a_1 = 4(1) + 1 = 5$$

$$a_2 = 4(2) + 1 = 9$$

Common difference,

$$d = a_2 - a_1 = 9 - 5 = 4$$

Thus (b) is correct option.

31. If a, b, c, d, e, f are in AP, then $e - c$ is equal to

(a) $2(c - a)$ (b) $2(d - c$

(c) $2(f - d)$ (d) $(d - c)$

Ans : (b) $2(d - c)$

Let x be the common difference of the AP a, b, c, d, e, f.

For an AP, $a_n = a + (n - 1)d$

$$e = a + (5 - 1)x$$

$$e = a + 4x \quad ...(1)$$

and $c = a + (3 - 1)x$

$$c = a + 2x \quad ...(2)$$

Using equation (1) and (2), we get

$$e - c = a + 4x - a - 2x$$

$$= 2x = 2(d - c)$$

Thus (b) is correct option.

32. If 7 times the 7th term of an AP is equal to 11 times its 11th term, then its term will be

(a) 7 (b) 11

(c) 18 (d) 0

Ans : (d) 0

In an AP, $a_n = a + (n - 1)d$

Now, according to the question,

$$7a_7 = 11a_{11}$$

$$7[a + (7 - 1)d] = 11[a + (11 - 1)d]$$

$$7(a + 6d) = 11(a + 10d)$$

$$7a + 42d = 11a + 110d$$

$$4a + 68d = 0$$

$$4(a + 17d) = 0$$

$$a + 17d = 0 \quad ...(1)$$

18th term of an AP,

$$a_{18} = a + (18 - 1)d = a + 17d$$

But from equation (1) this is zero.

33. The sum of 11 terms of an AP whose middle term is 30, is

(a) 320 (b) 330

(c) 340 (d) 350

Ans : (b) 330

Middle term is $\frac{11+1}{2} = 6$th term.

Now $a_n = a + (n - 1)d$

$$a_6 = a + 5d$$

$$30 = a + 5d$$

$$a = 30 - 5d$$

Now $S_n = \frac{n}{2}[2a + (n - 1)d]$

$$S_{11} = \frac{11}{2}(2a + 10d)$$

Substituting value of a we have

$$S_{11} = \frac{11}{2}[2(30-5d)+10d]$$

$$= \frac{11}{2}[60-10d+10d]$$

$$= 11 \times 30$$

$$S_{11} = 330$$

Thus (b) is correct option.

34. Five distinct positive integers are in a arithmetic progression with a positive common difference. If their sum is 10020, then the smallest possible value of the last term is

(a) 2002 (b) 2004

(c) 2006 (d) 2007

Ans : (c) 2006

Let the five integers be $a-2d$, $a-d$, a, $a+d$, $a+2d$.

Then, we have,

$(a-2d)+(a-d)+a+(a+d)+(a+2d) = 10020$

$$5a = 10020 \Rightarrow a = 2004$$

Now, as smallest possible value of d is 1.

Hence, the smallest possible value of $a+2d$ is 2004+2 = 2006

Thus (c) is correct option.

35. If the 2nd term of an AP is 13 and 5th term is 25, what is its 7th term?

(a) 30 (b) 33

(c) 37 (d) 38

Ans : (b) 33

We have $a_2 = 13$, and $a_5 = 25$

In an AP, $a_n = a+(n-1)d$

$$a_2 = a+(2-1)d = 13$$

$$a+d = 13 \quad ...(1)$$

and $a_5 = a+(5-1)d = 25$

$$a+4d = 25 \quad ...(2)$$

Subtracting equation (1) from equation (2), we get

$$3d = 25-13 = 12 \Rightarrow d = 4$$

From equation (1), $a = 13-4 = 9$

Now, 7th term, $a_7 = a+(7-1)d$

$$= 9+6\times 4 = 33$$

Thus (b) is correct option.

36. Assertion : Common difference of the AP -5, -1, 3, 7, ... is 4.

Reason : Common difference of the AP $a, a+d, a+2d,$.......... is given by $d = a_2 - a_1$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Common difference, $d = -1-(-5) = 4$

So, both A and R are correct and R explains A.

Thus (c) is correct option.

37. Assertion : Sum of first 10 terms of the arithmetic progression -0.5, -1.0, -1.5, ... is 31.

Reason : Sum of n terms of an AP is given as $S_n = \frac{n}{2}[2a+(n-1)d]$ where a is first term and d common difference.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

Assertion, S_{10}

$$= \frac{10}{2}[2(-0.5)+(10-1)(-0.5)]$$

$$= 5[-1-4.5]$$

$$= 5(-5.5) = 27.5$$

Assertion (A) is false but reason (R) is true.

Thus (d) is correct option.

38. Assertion : $a_n - a_{n-1}$ is not independent of n then the given sequence is an AP.

Reason : Common difference $d = a_n - a_{n-1}$ is constant or independent of n.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

Common difference of an AP $d = a_n - a_{n-1}$ is independent of n or constant.

So, A is correct but R is incorrect.

Thus (d) is correct option.

39. Assertion : If n^{th} term of an AP is $7-4n$, then its common differences is -4.

Reason : Common difference of an AP is given by $d = a_{n+1} - a_n$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

Assertion, $a_n = 7-4n$

$$d = a_{n+1} - a_n$$

$$= 7-4(n+1)-(7-4n)$$

$$= 7-4n-4-7+4n = -4$$

Both are correct. Reason is the correct explanation.

Thus (a) is correct option.

40. Assertion : If sum of the first n terms of an AP is given by $S_n = 3n^2 - 4n$. Then its n^{th} term is $a_n = 6n-7$.

Reason : n^{th} term of an AP, whose sum to n terms is S_n, is given by $a_n = S_n - S_{n-1}$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

nth term of an AP,

$$a_n = S_n - S_{n-1}$$

$$= 3n^2 - 4n - 3(n-1)^2 + 4(n-1)$$

$$= 6n - 7$$

So, both A and R are correct and R explains A.

Thus (a) is correct option.

41. In an AP, the letter d is generally used to denote the ....

Ans :

common difference

42. If a and d are respectively the first term and the common difference of an AP, $a+10d$, denotes the .... term of the AP.

Ans :

eleventh

43. An arithmetic progression is a list of numbers in which each term is obtained by .... a fixed number to the preceding term except the first term.

Ans :

adding

44. If S_n denotes the sum of n term of an AP, then $S_{12} - S_{11}$ is the .... term of the AP.

Ans :

twelfth

45. The nth term of an AP whose first term is a and common difference is d is ....

Ans :

$a+(n-1)d$

46. The nth term of an AP is always a .... expression.

Ans :

linear

47. The difference of corresponding terms of two AP's will be ....

Ans :

another AP

48. Fill the two blanks in the sequence 2 ...., 26, ... so that the sequence forms an AP.

Ans : [Board 2020 SQP Standard]

Let a and b be the two numbers. AP will be 2, a, 26, b.

Now, $$26 - a = a - 2$$

$$2a = 28 \Rightarrow a = \frac{28}{2} = 14$$

and $$b - 26 = 26 - a$$

$$a + b = 52$$

$$14 + b = 52 \Rightarrow b = 38$$

Thus $a = 14$ and $b = 38$.

Very Short Answer Questions

49. The sum of first 20 terms of the AP 1, 4, 7, 10 .... is

Ans : [Board 2020 Delhi Standard]

Given AP is 1, 4, 7, 10 ...

Here, $a = 1$, $d = 4 - 1 = 3$ and $n = 20$

$$S_{20} = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{20}{2}[2 \times 1 + (20-1)3]$$

$$= 10(2 + 57) = 10 \times 59 = 590$$

50. Show that $(a-b)^2$, (a^2+b^2) and $(a+b)^2$ are in AP.

Ans : [Board 2020 Delhi Standard]

Given, $(a-b)^2, (a^2+b^2)$ and $(a+b)^2$.

Common difference,

$$d_1 = (a^2+b^2) - (a-b)^2$$

$$= (a^2+b^2) - (a^2+b^2-2ab)$$

$$= a^2+b^2-a^2-b^2+2ab$$

$$= 2ab$$

and $$d_2 = (a+b)^2 - (a^2+b^2)$$

$$= a^2+b^2+2ab-a^2-b^2$$

$$= 2ab$$

Since, $d_1 = d_2$, thus, $(a-b)^2$, (a^2+b^2) and $(a+b)^2$ are in AP.

51. Find the sum of all 11 terms of an AP whose middle term is 30.

Ans : [Board 2020 OD Standard]

In an AP with 11 terms, the middle term is $\frac{11+1}{2} = 6^{th}$ term.

Now, $$a_6 = a + 5d = 30$$

Thus, $$S_{11} = \frac{11}{2}[2a + 10d]$$

$$= 11(a + 5d)$$

$$= 11 \times 30 = 330$$

52. If 4 times the 4^{th} term of an AP is equal to 18 times the 18^{th} term, then find the 22^{nd} term.

Ans : [Board 2020 Delhi Basic]

Let a be the first term and d be the common difference of the AP.

Now $$a_n = a + (n-1)d$$

As per the information given in question

$$4 \times a_4 = 18 \times a_{18}$$

$$4(a+3d) = 18(a+17d)$$

$$2a + 6d = 9a + 153d$$

$$7a = -147d$$

$$a = -21d$$

$$a + 21d = 0$$

$$a + (22-1)d = 0$$

$$a_{22} = 0$$

Hence, the 22^{nd} term of the AP is 0.

53. If the first three terms of an AP are b, c and $2b$, then find the ratio of b and c.

Ans : [Board 2020 SQP Standard]

Given, b, c and $2b$ are in AP.

Thus $$c - b = 2b - c$$

$$2c = 3b$$

$$\frac{2}{3} = \frac{b}{c}$$

$$\frac{b}{c} = \frac{2}{3} \Rightarrow b:c = 2:3$$

54. The n^{th} term of an AP is $(7-4n)$, then what is its common difference?

Ans : [Board 2020 Delhi Basic]

We have $a_n = 7 - 4n$

Putting $n = 1$, $a_1 = 7 - 4 = 3$

Putting $n = 2$, $a_2 = 7 - 8 = -1$

Common difference $d = a_2 - a_1$

$= -1 - 3 = -4$

55. In an AP, if the common difference $d = -4$, and the seventh term a_7 is 4, then find the first term.

Ans : [Board 2018]

We have $d = -4$

and $a_7 = 4$

Now $a_n = a + (n-1)d$

$a_7 = a + (7-1)d$

$4 = a + (7-1)(-4)$

$4 = a - 24 \Rightarrow a = 4 + 24 = 28$

First term of the AP is 28.

56. Find the sum of first 8 multiples of 3.

Ans : [Board 2018]

First 8 multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24 which are in AP where $a = 3$, $d = 3$ and $n = 8$.

Now $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_8 = \frac{8}{2}[2 \times 3 + (8-1)3]$

$= 4[6 + 21]$

$S_8 = 4 \times 27 = 108$

Thus, sum of first 8 multiples of 3 is 108.

57. Find, how many two digit natural numbers are divisible by 7.

Ans : [Board 2019 Delhi]

Two digits number which are divisible by 7 form an AP given by 14, 21, 28, ..., 98

Here, $a = 14$, $d = 21 - 14 = 7$ and $a_n = 98$

Now $a_n = a + (n-1)d$

$98 = 14 + (n-1)7$

$98 - 14 = 7n - 7$

$91 = 7n \Rightarrow n = 13$

Hence, there are 13 numbers divisible by 7.

58. Find the number of natural numbers between 102 and 998 which are divisible by 2 and 5 both.

Ans : [Board 2020 SQP Standard]

If any number is divisible by 2 and 5, it must be divisible by LCM of 2 and 5, i.e. 10.

Numbers between 102 .... 998 which are divisible by 2 and 5 are 110, 120, 130, ...990

Here $a = 110$, $d = 120 - 110 = 10$ and $a_n = 990$

$a_n = a + (n-1)d$

$990 = 110 + (n-1)10$

$880 = 10(n-1)$

$88 = n - 1$

$n = 88 + 1 = 89$

59. Is -150 a term of the AP 11, 8, 5, 2, ...?

Ans : [Board Term-2 2016]

Let the first term of an AP be a and common difference be d.

We have $a = 11$, $d = -3$, $a_n = -150$

Now $a_n = a + (n-1)d$

$-150 = 11 + (n-1)(-3)$

$-150 = 11 - 3n + 3$

$3n = 164$

or, $n = \frac{164}{3} = 54.66$

Since, 54.66 is not a whole number, -150 is not a term of the given AP

60. Which of the term of AP 5, 2, -1,...... is -49?

Ans : [Board Term-2 2012]

Let the first term of an AP be a and common difference d.

We have $a = 5$, $d = -3$

Now $a_n = a + (n-1)d$

Substituting all values we have

$$-49 = 5+(n-1)(-3)$$
$$-49 = 5-3n+3$$
$$3n = 49+5+3$$
$$n = \frac{57}{3} = 19^{th} \text{ term.}$$

61. Find the first four terms of an AP Whose first term is -2 and common difference is -2.

Ans : [Board Term-2 2012]

We have $a_1 = -2,$

$$a_2 = a_1 + d = -2+(-2) = -4$$
$$a_3 = a_2 + d = -4+(-2) = -6$$
$$a_4 = a_3 + d = -6+(-2) = -8$$

Hence first four terms are $-2, -4, -6, -8$

62. Find the tenth term of the sequence $\sqrt{2}, \sqrt{8}, \sqrt{18}$, ...

Ans : [Board Term-2 2016]

Let the first term of an AP be a and common difference be d.

Given AP is $\sqrt{2}, \sqrt{8}, \sqrt{18}$ or $\sqrt{2},\ 2\sqrt{2},\ 3\sqrt{2}$

where, $a = \sqrt{2}, d = \sqrt{2}, n = 10$

Now $a_n = a+(n-1)d$

$$a_{10} = \sqrt{2}+(10-1)\sqrt{2}$$
$$= \sqrt{2}+9\sqrt{2}$$
$$= 10\sqrt{2}$$

Therefore tenth term of the given sequence $\sqrt{200}$.

63. Find the next term of the series $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}$ ....

Ans : [Board Term-2 2012]

Let the first term of an AP be a and common difference d.

Here, $a = \sqrt{2},\ a+d = \sqrt{8} = 2\sqrt{2}$

$$d = 2\sqrt{2}-\sqrt{2} = \sqrt{2}$$

$$\text{Next term} = \sqrt{32}+\sqrt{2}$$
$$= 4\sqrt{2}+\sqrt{2}$$
$$= 5\sqrt{2}$$
$$= \sqrt{50}$$

64. Is series $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}$,.... an AP? Give rea

Ans :

Let common difference be d then we have

$$d = a_2 - a_1 = \sqrt{6}-\sqrt{3} = \sqrt{3}(\sqrt{2}-1)$$
$$d = a_3 - a_2 = \sqrt{9}-\sqrt{6} = 3-\sqrt{6}$$
$$d = a_4 - a_3 = \sqrt{12}-\sqrt{9} = 2\sqrt{3}-3$$

As common difference are not equal, the given series is not in AP

65. What is the next term of an AP $\sqrt{7},\ \sqrt{28},\ \sqrt{63}$,....?

Ans : [Board Term-2 Foreign 2014]

Let the first term of an AP be a and common difference be d.

Here, $a = \sqrt{7},\ a+d = \sqrt{28}$

$$d = \sqrt{28}-\sqrt{7} = 2\sqrt{7}-\sqrt{7}$$
$$= \sqrt{7}$$

$$\text{Next term} = \sqrt{63}+\sqrt{7}$$
$$= 3\sqrt{7}+\sqrt{7} = 4\sqrt{7}$$
$$= \sqrt{7\times 16}$$
$$= \sqrt{112}$$

66. If the common difference of an AP is -6, find $a_{16}-a_{12}$.

Ans : [Board Term-2 2014]

Let the first term of an AP be a and common difference be d.

Now $d = -6$

$$a_{16} = a+(16-1)(-6) = a-90$$
$$a_{12} = a+(12-1)(-6) = a-66$$
$$a_{16}-a_{12} = (a-90)-(a-66) = a-90-n+66$$
$$= -24$$

67. For what value of k will the consecutive terms $2k+1$, $3k+3$ and $5k-1$ form an AP?

Ans : [Board Term-2 Foreign 2016]

If x, y and z are in AP then we have

$$y-x = z-y$$

Thus if $2k+1,\ 3k+3,\ 5k-1$ are in AP then

$$(5k-1)-3k+3 = (3k+3)-(2k+1)$$
$$5k-1-3k-3\ \ 3k+3-2k-1$$
$$2k-4 = k+2$$
$$2k-k = 4+2$$
$$k = 6$$

68. Find the 25^{th} term of the AP $-5, \frac{-5}{2}, \frac{5}{2}, ...$

Ans : [Board Term-2 Foreign 2015]

Let the first term of an AP be a and common difference be d.

Here, $a = -5,\ d = -\frac{5}{2} - (-5) = \frac{5}{2}$

$$a_n = a + (n-1)d`$$

$$a_{25} = 5 + (25-1) \times \left(\frac{5}{2}\right)$$

$$= -5 + 60 = 55$$

69. The first three terms of an AP are $3y-1$, $3y+5$ and $5y+1$ respectively then find y.

Ans : [Board Term-2 Delhi 2015]

If x, y and z are in AP then we have

$$y - x = z - y$$

Therefore if $3y-1, 3y+5$ and $5y+1$ in AP

$$(3y+5) - (3y-1) = (5y+1) - (3\ \ +5$$

$$3y + 5 - 3y + 1 = 5y + 1 - 3y - 5$$

$$6 = 2y - 4$$

$$2y = 6 + 4$$

$$y = \frac{10}{2} = 5$$

70. For what value of k, $k+9$, $2k-1$ and $2k+7$ are the consecutive terms of an AP

Ans : [Board Term-2 OD 2016]

If x, y and z are consecutive terms of an AP then we have

$$y - x = z - y$$

Thus if $k+9, 2k-1,$ and $2k+7$ are consecutive terms of an AP then we have

$$(2k-1) - (k+9) = (2k+7) - (2\ \ -$$

$$2k - 1 - k - 9 = 2k + 7 - 2k + 1$$

$$k - 10 = 8\ \ k \Rightarrow 18$$

71. What is the common difference of an AP in which $a_{21} - a_7 = 84$?

Ans : [Board Term-2 2016]

Let the first term of an AP be a and common difference be d.

$$a_{21} - a_7 = 84$$

$$a + (21-1)d - [a + (7-1)d] = 84$$

$$a + 20d - a - 6d = 84$$

$$14d = 84$$

$$d = 6$$

72. In the AP $2, x, 26$ find the value of x.

Ans : [Board Term-2 2012]

If x, y and z are in AP then we have

$$y - x = z - y$$

Since $2, x$ and 26 are in AP we have

$$x - 2 = 26 - x$$

$$2x = 26 + 2$$

$$x = \frac{28}{2} = 14$$

73. For what value of k; $k+2$, $4k-6$, $3k-2$ are three consecutive terms of an AP.

Ans : [Board Term-2 Delhi 2014, 2012]

If x, y and z are three consecutive terms of an AP then we have

$$y - x = z - y$$

Since $k+2$, $4k-6$ and $3k-2$ are three consecutive terms of an AP, we obtain

$$(4k-6)-(k+2) = (3k-2)-(4 \;\; -6$$
$$4k-6-k-2 = 3k-2-4k+6$$
$$3k-8 = -k+4$$
$$4k = 4+8$$
$$k = \frac{12}{4} = 3$$

74. If $18, a, b, -3$ are in AP, then find $a+b$.

Ans : [Board Term-2 2012]

If $18, a, b, -3$ are in AP, then,

$$a-18 = -3-b$$
$$a+b = -3+18$$
$$a+b = 15$$

75. Find the common difference of the AP $\frac{1}{3q}, \frac{1-6q}{3q}, \frac{1-12q}{3q}, ...$

Ans : [Board Term-2 Delhi 2011]

Let common difference be d then we have

$$d = \frac{1-6q}{3q} - \frac{1}{3q}$$

$$= \frac{1-6q-1}{3q} = \frac{-6q}{3q} = -2$$

76. Find the first four terms of an AP whose first term is $3x+y$ and common difference is $x-y$.

Ans : [Board Term-2 2012]

Let the first term of an AP be a and common difference be d.

Now
$$a_1 = 3x+y$$
$$a_2 = a_1+d = 3x+y+x-y = 4x$$
$$a_3 = a_2+d = 4x+x-y = 5x-y$$
$$a_4 = a_3+d = 5x-y+x-y$$
$$= 6x-2y$$

So, the four terms are $3x+y$, $4x, 5x-y$ and $6x-2y$.

77. Find the 37^{th} term of the AP $\sqrt{x}$, $3\sqrt{x}$, $5\sqrt{x}$.

Ans : [Board Term-2 2012]

Let the nth term of an AP be a_n and common difference be d.

Here, $a_1 = \sqrt{x}$

$$a_2 = 3\sqrt{x}$$
$$d = a_2 - a_1 = 3\sqrt{x} - \sqrt{x} = 2\sqrt{x}$$
$$a_n = a+(n-1)d$$
$$a_{37} = \sqrt{x}+(37-1)2\sqrt{x}$$
$$= \sqrt{x}+36 \times 2\sqrt{x} = 73\sqrt{x}$$

78. For an AP, if $a_{25} - a_{20} = 45$, then find the value of d.

Ans : [Board Term-2 2011]

Let the first term of an AP be a and common difference be d.

$$a_{25} - a_{20} = \{a+(25-1)d\} - \{a+(20-1)d\}$$
$$45 = a+24d-a-19d$$
$$45 = 5d$$
$$d \; \frac{45}{5} = 9$$

79. Find the sum of first ten multiple of 5.

Ans : [Board Term-2 Delhi, 2014]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Here, $a = 5, n = 10, d = 5$

$$S_n = \frac{n}{2}[2a+(n-1)d$$
$$S_{10} = \frac{10}{2}[2 \times 5+(10-1)5]$$
$$= 5[10+9 \times 5]$$
$$= 5[10+45]$$
$$= 5 \times 55 = 275$$

Hence the sum of first ten multiple of 5 is 275.

80. Find the sum of first five multiples of 2.

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of $n$the term be S_n

Here, $a = 2, d = 2, n = 5$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$
$$S_5 = \frac{5}{2}[2 \times 2+(5-1)2]$$
$$= \frac{5}{2}[4+4 \times 2] = \frac{5}{2}[4+8]$$
$$= \frac{5}{2} \times 12 = 5 \times 6 = 30$$

81. Find the sum of first 16 terms of the AP 10, 6, 2, ...

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here, $a = 10, d = 6 - 1 = -4, n = 16$

$$S_n = \frac{n}{2}[2a + (n - \ )d$$

$$S_{16} = \frac{16}{2}[2 \times 10 + (16 - 1)(-4)]$$

$$= 8[20 + 15 \times (-4)]$$

$$= 8[20 - 60]$$

$$= 8 \times (-40)$$

$$= -320$$

82. What is the sum of five positive integer divisible by 6.

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of $n$the term be S_n

Here, $a = 6, d = 6, n = 5$

$$S_n = \frac{n}{2}[2a + (n-1)d$$

$$S_5 = \frac{5}{2}[2 \times 6 + (5-1)\ 6)$$

$$= \frac{5}{2}[12 + 4 \times 6]$$

$$= \frac{5}{2}[12 + 24] = \frac{5}{2}[36]$$

$$= 5 \times 18 = 90$$

83. If the sum of n terms of an AP is $2n^2 + 5n$, then find the 4^{th} term.

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Now, $S_n = 2n^2 + 5n$

n^{th} term of AP,

$$a_n = S_n - S_{n-1}$$

$$a_n = (2n^2 + 5n) - [2(n-1)^2 + 5(n-1)]$$

$$= 2n^2 + 5n - [2n^2 - 4n + 2 + 5n - 5]$$

$$= 2n^2 + 5n - 2n^2 - n + 3$$

$$= 4n + 3$$

Thus 4^{th} term $a_4 = 4 \times 4 + 3 = 19$

84. If the sum of first k terms of an AP is $3k^2 - k$ and its common difference is 6. What is the first term?

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n. Let the sum of k terms of AP is S_k.

We have $S_k = 3k^2 - k$

Now k^{th} term of AP,

$$a_k = S_k - S_{k-1}$$

$$a_k = (3k^2 - k) - [3(k-1)^2 - \ k - 1$$

$$= 3k^2 - k - [3k^2 - 6k + 3 - k + 1]$$

$$= 3k^2 - k - 3k^2 + 7k - 4$$

$$= 6k - 4$$

First term $a = 6 \times 1 - 4 = 2$

85. Which term of the AP $8, 14, 20, 26, ...$ will be 72 more than its 41^{st} term.

Ans : [Board Term-2 OD 2017]

Let the first term be a, common difference be d and nth term be a_n.

We have $a = 8, d = 6$.

Since n^{th} term is 72 more than 41^{st} term. we get

$$a_n = a_{41} + 72$$

$$8 + (n-1)6 = 8 + 40 \times 6 + 72$$

$$6n - 6 = 240 + 72$$

$$6n = 312 + 6 = 318$$

$$n = 53$$

86. If the n^{th} term of an AP $-1, 4, 9, 14, ...$ is 129. Find the value of n.

Ans : [Board Term-2 OD Compt. 2017]

Let the first term be a, common difference be d and nth term be a_n.

We have $a = -1$ and $d = 4 - (-1) = 5$

$$-1 + (n-1) \times 5 = a_n$$

$$-1 + 5n - 5 = 129$$

$$5n = 135$$

$$n = 27$$

Hence 27^{th} term is 129.

87. Write the n^{th} term of the AP $\frac{1}{m}, \frac{1+m}{m}, \frac{1+2m}{m}, ...$

Ans : [Board Term-2 OD Compt. 2017]

Let the first term be a, common difference be d and nth term be a_n.

We have $$a = \frac{1}{m}$$

$$d = \frac{1+m}{m} - \frac{1}{m} = 1$$

$$a_n = \frac{1}{m} + (n-1)1$$

Hence , $$a_n = \frac{1}{m} + n - 1$$

88. What is the common difference of an AP which $a_{21} - a_7 = 84$.

Ans : [Board Term-2 OD 2017]

Let the first term be a, common difference be d and nth term be a_n.

We have $$a_{21} - a_7 = 84$$

$$a + 20d - a - 6d = 84$$

$$14d = 84$$

$$d = \frac{84}{14} = 6$$

Hence common difference is 6.

89. Which term of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}....$ is the first negative.

Ans : [Board Term-2 OD 2017]

Let the first term be a, common difference be d and nth term be a_n.

We have $a = 20$ and $d = -\frac{3}{4}$

Let the n^{th} term be first negative term, then

$$a + (n-1)d < 0$$

$$20 + (n-1)\left(-\frac{3}{4}\right) < 0$$

$$20 - \frac{3}{4}n + \frac{3}{4} < 0$$

$$3n > 83$$

$$n > \frac{83}{3} = 27\frac{2}{3}$$

Hence 28^{th} term is first negative.

TWO MARKS QUESTIONS

90. If the sum of first m terms of an AP is the same as the sum of its first n terms, show that the sum of its first $(m+n)$ terms is zero.

Ans : [Board 2020 SQP Standard]

Let a be the first term and d be the common difference of the given AP. Then,

$$S_m = S_n$$

$$\frac{m}{2}\{2a + (m-1)d\} = \frac{n}{2}\{2a + (n- \ )d\}$$

$$2a(m-n) + \{m(m-1) - n(n-1)d\} = 0$$

$$2a(m-n) + [(m^2 - n^2) - (m-n)d] = 0$$

$$(m-n)[2a + (m+n-1)d] = 0$$

$$2a + (m+n-1)d = 0$$

Now, $$S_{m+n} = \frac{m+n}{2}\{2a + (m+n-1)d\}$$

$$= \frac{m+n}{2} \times 0 = 0$$

91. If $3k-2$, $4k-6$ and $k+2$ are three consecutive terms of AP, then find the value of k.

Ans : [Board 2020 OD Basic]

To be term of an AP the difference between two consecutive terms must be the same.

If $3k-2$, $4k-6$ and $k+2$ are terms of an AP, then

$$4k - 6 - (3k-2) = k + 2 - (4k-6)$$

$$4k - 6 - 3k + 2 = k + 2 - 4k + 6$$

$$k - 4 = 8 - 3k$$

$$4k = 12 \Rightarrow k = 3$$

Hence, the value of k is 3.

92. How many terms of AP 3, 5, 7, 9, ... must be taken to get the sum 120?

Ans : [Board 2020 OD Basic]

Given AP : 3, 5, 7, 9, ...

We have $a = 3$, $d = 2$ and $S_n = 120$

$$S_n = \frac{n}{2}[2a + (n- \ )d]$$

$$120 = \frac{n}{2}[2 \times 3 + (n-1)2]$$

$$120 = n(3 + n - 1)$$

$$120 = n(n+2)$$

$$n^2 + 2n - 120 = 0$$

$$n^2 + 12n - 10n - 120 = 0$$

$$(n+12)(n-10) = 0 \Rightarrow n = 10 \text{ or } n = -12$$

Neglecting $n = -12$ because n can't be negative we get $n = 10$. Hence, 10 terms must be taken to get the sum 120.

93. How many two digits numbers are divisible by 3?

Ans : [Board 2019 Delhi]

Numbers divisible by 3 are 3, 6, 9, 12, 15, ..., 96 and 99. Lowest two digit number divisible by 3 is 12 and highest two digit number divisible by 3 is 99.

Hence, the sequence start with 12, ends with 99 and common difference is 3.

So, the AP is 12, 15, 18, ..., 96, 99.

Here, $a = 12,\ d = 3$ and $a_n = 99$

$$a_n = a + (n-1)d$$

$$99 = 12 + (n-1)3$$

$$99 - 12 = 3(n-1)$$

$$n - 1 = \frac{87}{3} = 29 \Rightarrow n = 30$$

Therefore, there are 30, two digit numbers divisible by 3.

94. Which term of the AP 3, 15, 27, 39, ... will be 120 more than its 21st term?

Ans : [Board 2019 Delhi]

Given AP is 3, 15, 27, 39......

Here, first term, $a = 3$ and common difference, $d = 12$

Now, 21^{st} term of AP is

$$a_n = a + (n-1)d$$

$$a_{21} = 3 + (21-1) \times 12$$

$$= 3 + 20 \times 12 = 243$$

Therefore, 21^{st} term is 243.

Now we need to calculate term which is 120 more than 21^{st} term i.e it should be $243 + 120 = 363$

Therefore, $a_n = a + (n-1)d$

$$363 = 3 + (n-1)12$$

$$360 = 12(n-1)$$

$$n - 1 = 30 \Rightarrow n = 31$$

So, 31^{st} term is 120 more than 21^{st} term.

95. If S_n the sum of first n terms of an AP is given by $S_n = 3n^2 - 4n$, find the n^{th} term.

Ans : [Board 2019 Delhi]

We have $S_n = 3n^2 - 4n$

Substituting $n = 1$, we get

$$S_1 = 3 \times 1^2 - 4 \times 1 = -1$$

So, sum of first term of AP is -1, but sum of first term is the first term itself,

Thus first term $a_1 = -1$

Now substituting $n = 2$ we have

$$S_2 = 3 \times 2^2 - 4 \times 2 = 4$$

Sum of first two terms is 4.

$$a_1 + a_2 = 4$$

$$-1 + a_2 = 4 \Rightarrow a_2 = 5$$

Hence, common difference,

$$d = a_2 - a_1 = 5 - (-1) = 6$$

Now n^{th} term, $a_n = a_1 + (n-1)d$

, $a_n = -1 + (n-1)6$

$$a_n = 6n - 7$$

Therefore, n^{th} term is $6n - 7$.

96. Find the 21^{st} term of the AP $-4\frac{1}{2}, -3, -1\frac{1}{2}, ...$

Ans : [Board 2019 OD]

Given AP is $-4\frac{1}{2}, -3, -1\frac{1}{2}, ...$ or $-\frac{9}{2}, -3, -\frac{3}{2}, ...$

First term, $a = \frac{-9}{2}$

Common difference,

$$d = -3 - \left(-\frac{9}{2}\right) = -3 + \frac{9}{2}$$

$$= \frac{-6+9}{2} = \frac{3}{2}$$

Now $a_n = a + (n-1)d$

$$a_{21} = \left(-\frac{9}{2}\right) + (21-1)\left(\frac{3}{2}\right)$$

$$= -\frac{9}{2} + 20 \times \frac{3}{2} = -\frac{9}{2} + 30$$

$$= \frac{-9+30}{2} = \frac{51}{2} = 25\frac{1}{2}$$

Hence, 21^{st} term of given AP is $25\frac{1}{2}$.

97. If the sum of first n terms of an AP is n^2, then find its 10th term.

Ans : [Board 2019 Delhi]

We have $S_n = n^2$ ...(1)

Substituting $n = 1$ in equation (1), we have

$$S_1 = 1$$

Hence, sum of first term of AP is 1, but sum of first term is first term itself.

So, first term, $a = 1$ ...(2)

Substituting $n = 2$ in equation (1), we have

$$S_2 = (2)^2 = 4$$

Sum of first 2 terms is 4.

Now $a + a_2 = 4$ ...(3)

From equation (2) and (3) we have

$$a_2 = 3$$

Now, common difference,

$$d = a_2 - a = 3 - 1 = 2$$

Now, 10^{th} term of AP,

$$a_{10} = a + (10-1)d$$

$$= 1 + 9 \times 2 = 19$$

Hence, the 10^{th} term of AP is 19.

98. Is 184 a term of the sequence $3, 7, 11, ...$?

Ans : [Board Term-2 2012]

Let the first term of an AP be a, common difference be d and number of terms be n.

Let $a_n = 184$

Here, $a = 3, d = 7 - 3 = 11 - 7 = 4$

Now $a_n = a + (n-1)d,$

$$184 = 3 + (n-1)4$$

$$\frac{181}{4} = n - 1$$

$$45.25 = n - 1$$

$$46.25 = n$$

Since 46.25 is not an whole number, thus 184 is not a term of given AP

99. Find, 100 is a term of the AP $25, 28, 31, ...$ or not.

Ans : [Board Term-2 2012]

Let the first term of an AP be a, common difference be d and number of terms be n.

Let $a_n = 100$

Here $a = 25, d = 28 - 25 = 31 - 28 = 3$

Now $a_n = a + (n-1)d,$

$$100 = 25 + (n-1) \times 3$$

$$100 - 25 = 75 = (n-1) \times 3$$

$$25 = n - 1$$

$$n = 26$$

Since 26 is an whole number, thus 100 is a term of given AP.

100. Find the 7^{th} term from the end of AP $7, 10, 13, ... 184$.

Ans : [Board Term-2 2012]

Let us write AP in reverse order i.e., $184, ... 13, 10, 7$

Let the first term of an AP be a and common difference be d.

Now $d = 7 - 10 = -3$

$$a = 184, \ n = 7$$

7^{th} term from the original end,

$$a_7 = a + 6d$$

$$a_7 = 184 + 6(-3)$$

$$= 184 - 18 = 166.$$

Hence, 166 is the 7^{th} term from the end.

101. Which term of an AP 150, 147, 144, ... is its first negative term?

Ans : [KVS 2014]

Let the first term of an AP be a, common difference be d and nth term be a_n.

For first negative term $a_n < 0$

$$a + (n-1)d < 0$$

$$150 + (n-1)(-3) < 0$$

$$150 - 3n + 3 < 0$$

$$-3n < -153$$

$$n > 51$$

Therefore, the first negative term is 52^{nd} term.

102. In a certain AP 32^{th} term is twice the 12^{th} term. Prove that 70^{th} term is twice the 31^{st} term.

Ans : [Board Term-2 2015, 2012]

Let the first term of an AP be a, common difference be d and nth term be a_n.

Now we have
$$a_{32} = 2a_{12}$$
$$a+31d = 2(a+11d)$$
$$a+31d = 2a+22d$$
$$a = 9d$$
$$a_{70} = a+69d$$
$$= 9d+69d = 78d$$
$$a_{31} = a+30d$$
$$= 9d+30d = 39d$$
$$a_{70} = 2a_{31}$$
Hence Proved.

103. The 8^{th} term of an AP is zero. Prove that its 38^{th} term is triple of its 18^{th} term.

Ans : [Board Term-2 2012]

Let the first term of an AP be a, common difference be d and nth term be a_n.

We have, $a_8 = 0$ or, $a+7d = 0$ or, $a = -7$

Now
$$a_{38} = a+37d$$
$$a_{38} = -7d+37d = 30d$$
$$a_{18} = a+17d$$
$$= -7d+17d = 10d$$
$$a_{38} = 30d = 3\times 10d = 3\times a_{18}$$
$$a_{38} = 3a_{18}$$
Hence Proved

104. If five times the fifth term of an AP is equal to eight times its eighth term, show that its 13^{th} term is zero.

Ans : [Board Term-2 2012]

Let the first term of an AP be a, common difference be d and nth term be a_n.

Now
$$5a_5 = 8a_8$$
$$5(a+4d) = 8(a+7d)$$
$$5a+20d = 8a+56d$$
$$3a+36d = 0$$
$$3(a+12d) = 0$$
$$a+12d = 0$$
$$a_{13} = 0$$
Hence Proved

105. The fifth term of an AP is 20 and the sum of its seventh and eleventh terms is 64. Find the common difference.

Ans : [Board Term-2 Foreign 2015]

Let the first term be a and common difference be d.

$$a+4d = 20 \qquad ...(1)$$
$$a+6d+a+10d = 64$$
$$a+8d = 32 \qquad ...(2)$$

Solving equations (1) and (2), we have
$$d = 3$$

106. The ninth term of an AP is -32 and the sum of its eleventh and thirteenth term is -94. Find the common difference of the AP

Ans : [Board Term-2 Foreign 2015]

Let the first term be a and common difference be d.

Now
$$a+8d = a_9$$
$$a+8d = -32 \qquad ...(1)$$
and
$$a_{11}+a_{13} = -94$$
$$a+10d+a+12d = -94$$
$$a+11d = -47 \qquad ...(2)$$

Solving equation (1) and (2), we have
$$d = -5$$

107. The seventeenth term of an AP exceeds its 10^{th} term by 7. Find the common difference.

Ans : [Board Term-2 2015, 2014]

Let the first term be a and common difference be d.

Now
$$a_{17} = a_{10}+7$$
$$a+16d = a+9d+7$$
$$16d-9d = 7$$
$$7d = 7$$
$$d = 1$$

Thus common difference is 1.

108. The fourth term of an AP is 11. The sum of the fifth and seventh terms of the AP is 34. Find the c difference.

Ans :

Let the first term be a and common difference be d.

Now $a_4 = 11$

$$a + 3d = 11 \quad ...(1)$$

and $a_5 + a_7 = 34$

$$a + 4d + a + 6d = 34$$

$$, \quad 2a + 10d = 34$$

$$, \quad a + 5d = 17 \quad ...(2)$$

Solving equations (1) and (2) we have

$$d = 3$$

109. Find the middle term of the AP 213, 205, 197, ... 37.

Ans : [Board Term-2 Delhi 2015]

Let the first term of an AP be a, common difference be d and number of terms be m.

Here, $a = 213, d = 205 - 213 = -8, a_m = 37$

$$a_m = a + (m-1)d$$

$$37 = 213 + (m-1)(-8)$$

$$37 - 213 = -8(m-1)$$

$$m - 1 = \frac{-176}{-8} = 22$$

$$m = 22 + 1 = 23$$

The middle term will be $= \frac{23+1}{2} = 12^{th}$

$$a_{12} = a + (12-1)d$$

$$= 213 + (12-1)(-8)$$

$$= 213 - 88 = 125$$

Middle term will be 125.

110. Find the middle term of the AP 6, 13, 20, ... 216.

Ans : [Board Term-2 Delhi 2015]

Let the first term of an AP be a, common difference be d and number of terms be m.

Here, $a = 6, a_m = 216, d = 13 - 6 = 7$

$$a_m = a + (m-1)d$$

$$216 = 6 + (m-1)(7)$$

$$216 - 6 = 7(m-1)$$

$$m - 1 = \frac{210}{7} = 30$$

$$m = 30 + 1 = 31$$

The middle term will be $= \frac{31+1}{2} = 16^{th}$

$$a_{16} = a + (16-1)d$$

$$= 6 + (16-1)(7)$$

$$= 6 + 15 \times 7$$

$$= 6 + 105 = 111$$

Middle term will be 111.

111. If the 2^{nd} term of an AP is 8 and the 5^{th} term is 17, find its 19^{th} term.

Ans : [Board Term-2 2016]

Let the first term be a and common difference be d.

Now $a_2 = a + d$

$$8 = a + d \quad (1)$$

and $a_5 = a + 4d$

$$17 = a + 4d \quad (2)$$

Solving (1) and (2), we have

$$a = 5, d = 3,$$

$$a_{19} = a + 18d$$

$$= 5 + 54 = 59$$

112. If the number $x+3, 2x+1$ and $x-7$ are in AP find the value of x.

Ans : [Board Term-2 2012]

If x, y and z are three consecutive terms of an AP then we have

$$y - x = z - y$$

$$(2x+1) - (x+3) = (x-7) - (2x+1$$

$$2x + 1 - x - 3 = x - 7 - 2x - 1$$

$$x - 2 = -x - 8$$

$$2x = -6$$

$$x = -3$$

113. Find the values of a, b and c, such that the numbers a, 10, b, c, 31 are in AP

Ans : [Board Term-2 2012]

Let the first term be a and common difference be d.

Since $a, 10, b, c, 31$ are in AP, then

$$a + d = 10 \quad (1)$$

$$a + 4d = a_5$$

$$a + 4d = 31 \quad (2)$$

Solving (1) and (2) we have

$$d = 7 \text{ and } a = 3$$

Now $a = 3, b = 3 + 14 = 17, c = 3 + 21 = 24$

Thus $a = 3, b = 17, c = 24$.

114. For AP show that $a_p + a_{p+2q} = 2a_{p+q}$.

Ans : [Board Term-2 2012]

Let the first term be a and the common difference be d. Let a_n be the nth term.

$$a_p = a + (p-1)d$$

$$a_{p+2q} = a + (p+2q-1)d$$

$$a_p + a_{p+2q} = a + (p-1)d + a + (p+2q-1)d$$

$$= a + pd - d + a + pd + 2qd - d$$

$$= 2a + 2pd + 2qd - 2d$$

or $a_p + a_{p+2q} = 2[a + (p+q-1)d]$ ...(1)

But $2a_{p+q} = 2[a + (p+q-1)d]$ ...(2)

From (1) and (2), we get $a_p + a_{p+2q} = 2a_{p+q}$

115. The sum of first terms of an AP is given by $S_n = 2n^2 + 8n$. Find the sixteenth term of the AP.

Ans : [Board SQP 2017]

Let the first term be a, common difference be d and nth term be a_n.

Now $S_n = 2n^2 + 3n$

$$S_1 = 2 \times 1^2 + 3 \times 1 = 2 + 3 = 5$$

Since $S_1 = a_1$,

$$a_1 = 5$$

$$S_2 = 2 \times 2^2 + 3 \times 2 = 8 + 6 = 14$$

$$a_1 + a_2 = 14$$

$$a_2 = 14 - a_1 = 14 - 5 = 9$$

$$d = a_2 - a_1 = 9 - 5 = 4$$

$$a_{16} = a + (16-1)\,d$$

$$= 5 + 15 \times 4 = 65$$

116. The 4^{th} term of an AP is zero. Prove that the 25^{th} term of the AP is three times its 11^{th} term.

Ans : [Board Term-2 OD 2016]

Let the first term be a, common difference be d and nth term be a_n.

We have, $a_4 = 0$

$$a + 3d = 0 \quad [\,a + (n-1)d = a_n]$$

$$3d = -a$$

$$-3d = a \quad ...(1)$$

Now, $a_{25} = a + 24d = -3d + 24d = 21d$ ...(2)

$a_{11} = a + 10d = -3d + 10d = 7d$ ...(3)

From equation (2) and (3) we have

$a_{25} = 3a_{11}$ Hence Proved.

117. How many terms of the AP $65, 60, 55, ...$ be taken so that their sum is zero?

Ans : [Board Term-2 Delhi 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a = 65, d = -5, S_n = 0$

Now $S_n = \frac{n}{2}[2a + (n-1)d$

Let sum of n term be zero, then we have

$$\frac{n}{2}[130 + (n-1)(-5)] = 0$$

$$\frac{n}{2}[130 + 5n + 5] = 0$$

$$135n - 5n^2 = 0$$

$$n(135 - 5n) = 0$$

$$5n = 135$$

$$n = 27$$

118. How many terms of the AP $18, 16, 14 .....$ be taken so that their sum is zero?

Ans : [Board Term-2 Delhi 2016]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Here $a = 18, d = -2, S_n = 0$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Let sum of n term be zero, then we have

$$\frac{n}{2}[36+(n-1)(-2)] = 0$$

$$n(38-2n) = 0$$

$$n = 19$$

119. How many terms of the AP $27, 24, 21 ....$ should be taken so that their sum is zero?

Ans : [Board Term-2 Delhi 2016]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Here $a = 27, d = -3, S_n = 0$

$$S_n = \frac{n}{2}[2a+(n-1)d$$

Let sum of n term be zero, then we have

$$\frac{n}{2}[54+(n-1)(-3)] = 0$$

$$n(-3n+57) = 0$$

$$n = 19$$

120. In an AP, if $S_5 + S_7 = 167$ and $S_{10} = 235$, then find the AP, where S_n denotes the sum of first n terms.

Ans : [Board Term-2 OD 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$S_n = \frac{n}{2}[2a+(n-1)d$$

$$S_5 + S_7 = 167$$

$$\frac{5}{2}(2a+4d)+\frac{7}{2}(2a+6d) = 167$$

$$5a+10d+7a+21d = 167$$

$$12a+31d = 167 \quad ...(1)$$

Now we have $S_{10} = 235$, thus

$$\frac{10}{2}[2a+(10-1)d] = 235$$

$$5(2a+9d) = 235$$

$$2a+9d = 47 \quad (2)$$

Solving (1) and (2), we get

$$a = 1, d = 5$$

Thus AP is 1, 6, 11....

121. Find the sum of sixteen terms of an AP $-1, -5, -9, ...$

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Here, $a_1 = -1, a_2 = -5$ and $d = -4$

Now $$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$S_{16} = \frac{16}{2}[2\times(-1)+(16-1)(-4)]$$

$$= 8[-2-60] = 8(-62)$$

$$= -496$$

122. If the n^{th} term of an AP is $7-3n$, find the sum of twenty five terms.

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here $n = 25, a_n = 7-3n$

Taking $n = 1,2,3,....$ we have

$$a_1 = 7-3\times1 = 4$$

$$a_2 = 7-3\times2 = 1$$

$$a_3 = 7-3\times3 = -2$$

Thus required AP is $4, 1, -2, ...$

Here, $a = 4, d = 1-4 = -3$

Now, $$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$= \frac{25}{2}[2\times4+(25-1)(-3)]$$

$$= \frac{25}{2}[8+24(-3)]$$

$$= \frac{25}{2}(8-72) = -800$$

123. If the 1^{st} term of a series is 7 and 13^{th} term is 35. Find the sum of 13 terms of the sequence.

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Here $a = 7, a_{13} = 35$

$$a_n = a+(n-1)d$$
$$a_{13} = a+12d$$
$$35 = 7+12d \Rightarrow d = \frac{7}{3}$$

Now $$S_n = \frac{n}{2}[2a+(n- \ )d$$
$$S_{13} = \frac{13}{2}\left[2\times 7+12\times\left(\frac{7}{3}\right)\right]$$
$$= \frac{13}{2}[14+28]$$
$$= \frac{13}{2}\times 42 = 273$$

124. If the n^{th} term of a sequence is $3-2n$. Find the sum of fifteen terms.

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here, $a_n = 3-2n$

Taking $n=1$, $a_1 = 3-2 = 1$

15th term, $a_{15} = 3-2\times 15 = 3-30 = -27$

Now $$S_n = \frac{n}{2}(a+1)$$
$$S_{15} = \frac{15}{2}[1+(-27)$$
$$= \frac{15}{2}[-26]$$
$$= 15\times(-13) = -195$$

125. If S_n denotes the sum of n terms of an AP whose common difference is d and first term is a, find $S_n - 2S_{n-1}+S_{n-2}$.

Ans : [Board Term-2 2011]

We have $$a_n = S_n - S_{n-1}$$
$$a_{n-1} = S_{n-1}-S_{n-2}$$
$$S_n - 2S_{n-1}+S_{n-2} = S_n - S_{n-1} - S_{n-1} + S_{n-2}$$
$$= (S_n - S_{n-1}) - (S_{n-1}-S_{n-2})$$
$$= a_n - a_{n-1} = d$$

126. The sum of first n terms of an AP is $5n-n^2$. Find the n^{th} term of the AP

Ans : [Board Term-2 Foreign 2014]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have, $S_n = 5n - n^2$

Now, n^{th} term of AP,
$$a_n = S_n - S_{n-1}$$
$$= (5n-n^2) - [5(n-1)-(n-1)^2]$$
$$= 5n - n^2 - [5n-5-(n^2+1-2n)]$$
$$= 5n-n^2-(5n-5-n^2-1+2n)$$
$$= 5n-n^2-7n+6+n^2$$
$$= -2n+6$$
$$a_n = -2(n-3)$$

Thus n^{th} term is $= -2(n-3)$

127. The first and last term of an AP are 5 and 45 respectively. If the sum of all its terms is 400, find its common difference.

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a = 5, a_n = 45$

Now $$45 = 5+(n-1)d$$
$$(n-1)d = 40 \quad ...(1)$$

Given, $$S_n = 400$$

Now $$S_n = \frac{n}{2}(a+a_n)$$
$$400 = \frac{n}{2}(5+45)$$
$$800 = 50n$$
$$n = 16$$

Substituting this value of n in (1) we have
$$(n-1)d = 40$$
$$15d = 40$$
$$d = \frac{40}{15} = \frac{8}{3}$$

128. If the sum of the first 7 terms of an AP is 49 and that of the first 17 terms is 289, find the sum of its first n terms.

Ans : [Board Term-2 Foreign 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$S_n = \frac{n}{2}[2a+(n-1)d$$

Now $$S_7 = \frac{7}{2}(2a+6d) = 49$$

$$a+3d = 7 \quad ...(1)$$

and $$S_{17} = \frac{17}{2}(2a+16d) = 289$$

$$a+8d = 17$$

Subtracting (1) from (2), we get

$$5d = 10 \Rightarrow d = 2$$

Substituting this value of d in (1) we have

$$a = 1$$

Now $$S_n = \frac{n}{2}[2\times 1+(n-1)2]$$

$$= \frac{n}{2}[2+2n-2] = n^2$$

Hence, sum of n terms is n^2.

129. How many terms of the AP $-6, \frac{-11}{2}, -5, -\frac{9}{2}....$ are needed to give their sum zero.

Ans : [Board Term-2 OD Compt. 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a = -6, d = -\frac{11}{2} - (-6) = \frac{1}{2}$

$$S_n = \frac{n}{2}[2a+(n-1)d$$

Let sum of n term be zero, then we have

$$\frac{n}{2}\left[2\times -6+(n-1)\frac{1}{2}\right] = 0$$

$$\frac{n}{2}\left[-12+\frac{n}{2}-\frac{1}{2}\right] = 0$$

$$\frac{n}{2}\left[\frac{n}{2}-\frac{25}{2}\right] = 0$$

$$n^2-25n = 0$$

$$n(n-25) = 0$$

$$n = 25$$

Hence 25 terms are needed.

130. Which term of the AP $3, 12, 21, 30, ...$ will be 90 more than its 50^{th} term.

Ans : [Board Term-2 Compt. 2017]

Let the first term be a, common difference be d and nth term be a_n.

We have $$a = 3, d = 9$$

Now $$a_n = a+(n-1)d$$

$$a_{50} = 3+49\times 9 = 444$$

Now, $$a_n - a_{50} = 90$$

$$3+(n-1)9-444 = 90$$

$$(n-1)9 = 90+441$$

$$(n-1) = \frac{531}{9} = 49$$

$$n = 49+1 = 50$$

131. The 10^{th} term of an AP is -4 and its 22^{nd} term is -16. Find its 38^{th} term.

Ans : [Board Term-2 Delhi Compt. 2017]

Let the first term be a, common difference be d and nth term be a_n.

$$a_{10} = a+9d = -4 \quad (1)$$

and $$a_{22} = a+21d = -16 \quad (2)$$

Subtracting (2) from (1) we have

$$12d = -12 \Rightarrow d = -16$$

Substituting this value of d in (1) we get

$$a = 5$$

Thus $$a_{38} = 5+37\times -1 = -32$$

Hence, $$a_{38} = -32$$

132. Find how many integers between 200 and 500 are divisible by 8.

Ans : [Board Term-2 Delhi Compt. 2017]

Number divisible by 8 are 208, 2016, 224, ... 496. It is an AP

Let the first term be a, common difference be d and nth term be a_n.

We have $a = 208, d = 8$ and $a_n = 496$

Now $$a+(n-1)d = a_n$$

$$208+(n-1)d = 496$$

$$(n-1)8 = 496-208$$

$$n-1 = \frac{288}{8} = 36$$

$$n = 36+1 = 37$$

Hence, required numbers divisible by 8 is 37.

133. The fifth term of an AP is 26 and its 10^{th} term is 51. Find the AP

Ans : [Board Term-2 OD Compt. 2017]

Let the first term be a, common difference be d and nth term be a_n.

$$a_5 = a+4d = 26 \quad ...(1)$$

$$a_{10} = a+9d = 51 \quad ...(2)$$

Subtracting (1) from (2) we have

$$5d = 25 \Rightarrow d = 5$$

Substituting this value of d in equation (1)

we get

$$a = 6$$

Hence, the AP is 6, 11, 16, ...

134. Find the AP whose third term is 5 and seventh term is 9.

Ans : [Board Term-2 Delhi Compt. 2017]

Let the first term be a, common difference be d and nth term be a_n.

Now $$a_3 = a+2d = 5 \quad ...(1)$$

and $$a_7 = a+6d = 9 \quad ...(2)$$

Subtracting (2) from (1) we have

$$4d = 4 \Rightarrow d = 1$$

Substituting this value of d in (1) we get

$$a = 3$$

Hence AP is 3, 4, 5, 6, ...

135. Find whether -150 is a term of the AP $11, 8, 5, 2, ...$

Ans : [Board Term-2 Delhi Compt. 2017]

Let the first term be a, common difference be d and nth term be a_n.

Let the n^{th} term of given AP $11, 8, 5, 2, ...$ be -150

Hence $a = 11$, $d = 8-11 = -3$ and $a_n = -150$

$$a+(n-1)d = a_n$$

$$11+(n-1)(-3) = -150$$

$$(n-1)(-3) = -161$$

$$(n-1) = \frac{-161}{-3} = 53\frac{2}{3}$$

which is not a whole number. Hence -150 is not a term of given AP.

136. If seven times the 7^{th} term of an AP is equal to eleven times the 11^{th} term, then what will be its 18^{th} term.

Ans : [Board Term-2 Foreign 2017]

Let the first term be a, common difference be d and nth term be a_n.

$$7a_7 = 11a_{11}$$

Now $$7(a+6d) = 11(a+10d)$$

$$7a+42d = 11a+110d$$

$$11a-7a = 42d-110d$$

, $$4a = -68d$$

$$4a+68d = 0$$

$$4(a+17d) = 0$$

$$a+17d = 0$$

Hence, $$a_{18} = 0$$

137. In an AP of 50 terms, the sum of the first 10 terms is 210 and the sum of its last 15 terms is 2565. Find the AP

Ans : [Board Term-2 Foreign 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$S_{10} = 210$$

$$S_n = \frac{n}{2}[2a+(n-1)d$$

$$210 = \frac{10}{2}(2a+9d)$$

$$42 = 2a+9d \quad (1)$$

Now $$a_{36} = a+35d$$

$$a_{50} = a+49d$$

Sum of last 15 terms,

$$S_{36-50} = \frac{n}{2}(a_{36}+a_{50})$$

$$2565 = \frac{15}{2}(a+35d+a+49d)$$

$$171 = \frac{1}{2}(2a+84d)$$

$$171 = a+42d \quad (2)$$

Solving (1) and (2) we get $a = 3$ and $d = 4$

Hence, AP is 3, 7, 11, ...

THREE MARKS QUESTIONS

138. The sum of four consecutive number in AP is 32 and the ratio of the product of the first and last term to the product of two middle terms is 7 : 15. Find the numbers.

Ans : [Board 2020 Delhi Standard, 2018]

Let the four consecutive terms of AP be $(a-3d)$, $(a-d)$, $(a+d)$ and $(a+3d)$.

As per question statement we have

$$a-3d+a-d+a+d+a+3d = 32$$

$$4a = 32 \Rightarrow a = 8$$

and $$\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$$

$$\frac{a^2-9d^2}{a^2-d^2} = \frac{7}{15}$$

$$\frac{64-9d^2}{64-d^2} = \frac{7}{15}$$

$$960-135d^2 = 448-7d^2$$

$$7d^2-135d^2 = 448-960$$

$$-128d^2 = -512$$

$$d^2 = 4 \Rightarrow d = \pm 2$$

Hence, the number are 2, 6, 10 and 14 or 14, 10, 6 and 2.

139. The sum of the first 7 terms of an AP is 63 and that of its next 7 terms is 161. Find the AP.

Ans : [Board 2020 Delhi Standard]

We have $$S_7 = 63$$

Now $$S_n = \frac{n}{2}[2a+(n- \ )d]$$

$$63 = \frac{7}{2}[2a+6d]$$

$$9 = a+3d \quad ...(1)$$

Now, sum of next 7 terms,

$$S_{8-14} = 161$$

$$S_{8-14} = \frac{7}{2}(a_8+a_{14})$$

$$161 = \frac{7}{2}(a+7d+a+13d)$$

$$161 = \frac{7}{2}(2a+20d)$$

$$23 = a+10d \quad ...(2)$$

Subtracting equation (1) from (2) we have

$$14 = 7d \Rightarrow d = 2$$

Substituting the value of d in (1), we get

$$a = 3$$

Hence, the AP is 3, 5, 7, 9, ...

140. Which term of the AP 20, $19\frac{1}{4}$, $18\frac{1}{2}$, $17\frac{3}{4}$, ... is the first negative term.

Ans : [Board 2020 OD Standard]

Here, $$a = 20$$

and $$d = \frac{77}{4}-20 = -\frac{3}{4}$$

Let a_n is the first negative term, thus $a_n < 0$.

Now $$a_n = a+(n-1)d$$

$$20+(n-1)\left(-\frac{3}{4}\right) < 0$$

$$80-3n+3 < 0$$

$$83-3n < 0$$

$$n > \frac{83}{3} \quad n > 27.6$$

$$n = 28$$

Hence, the first negative term is 28th term.

141. Find the middle term of the AP 7, 13, 19, ..., 247.

Ans : [Board 2020 OD Standard]

In this AP, $$a = 7$$

$$d = 13-7 = 6$$

$$a_n = a+(n-1)d$$

$$247 = 7+(n-1)6$$

$$6(n-1) = 240$$

$$n-1 = 40 \Rightarrow n = 41$$

Hence, the middle term $= \frac{n+1}{2} = \frac{41+1}{2} = \frac{42}{2} = 21.$

$$a_{21} = 7+(21-1)6 = 127$$

142. Show that the sum of all terms of an AP whose first term is a, the second term is b and last term is c, is equal to $\dfrac{(a+c)(b+c-2a)}{2(b-a)}$

Ans : [Board 2020 OD Standard]

Given, first term, $A = a$

and second term $A_2 = b$

Common difference, $D = b-a$

Last term, $A_n = c$

$$A+(n-1)d = c$$
$$a+(n-1)(b-a) = c$$
$$(b-a)(n-1) = c-a$$
$$n-1 = \frac{c-a}{b-a}$$
$$n = \frac{c-a}{b-a}+1$$
$$= \frac{c-a+b-a}{b-a}$$
$$n = \frac{b+c-2a}{b-a}$$

Now sum of all terms

$$S_n = \frac{n}{2}[A+A_n] = \frac{(b+c-2a)}{2(b-a)}[a+c]$$
$$= \frac{(a+c)(b+c-2a)}{2(b-a)}$$ Hence Proved

143. If in an AP, the sum of first m terms is n and the sum of its first n terms is m, then prove that the sum of its first $(m+n)$ terms is $-(m+n)$.

Ans : [Board 2020 OD Standard]

Let 1st term of series be a and common difference be d, then we have

$$S_m = n$$

and $$S_n = m$$

$$\frac{m}{2}[2a+(m-1)d] = n \quad ...(1)$$
$$\frac{n}{2}[2a+(n-1)d] = m \quad ...(2)$$

Subtracting we have

$$a(m-n)+\frac{d}{2}[m(m-1)-n(n-1)] = n-m$$
$$2a(m-n)+d[m^2-n^2-(m-n)] = 2(n-m)$$
$$2a(m-n)+d(m-n)[(m+n)-1] = 2(n-m)$$
$$2a+d[(m+n)-1] = -2$$

Now, $$S_{m+n} = \frac{m+n}{2}[2a+(m+n-1)d]$$
$$= \frac{m+n}{2}(-2)$$
$$= -(m+n)$$

144. The 17th term of an AP is 5 more than twice its 8th term. If 11th term of AP is 43, then find its n^{th} term.

Ans : [Board 2020 OD Basic]

Let a be the first term and d be the common difference.

n^{th} term of an AP,

$$a_n = a+(n-1)d$$

Since 17th term of an AP is 5 more than twice of its 8th term, thus

$$a+(17-1)d = 5+2[a+(8-1)\ ]$$
$$a+16d = 5+2(a+7d)$$
$$a+16d = 5+2a+14d$$
$$2d-a = 5 \quad ...(1)$$

Since 11th term of AP is 43,

$$a+(11-1)d = 43$$
$$a+10d = 43 \quad ...(2)$$

Solving equation (1) and (2), we have

$$a = 3 \text{ and } d = 4$$

Hence, n^{th} term would be

$$a_n = 3+(n-1)4 = 4n-1$$

145. How many terms of the AP 24, 21, 18, ... must be taken so that their sum is 78?

Ans : [Board 2020 Delhi Basic]

Given : 24, 21, 18, ... are in AP.

Here, $a = 24,\ d = 21-24 = -3$

Sum of n term, $S_n = \frac{n}{2}[2a+(n-\ \ )d]$

$$78 = \frac{n}{2}[2\times 24+(n-1)(-3)]$$
$$156 = n(48-3n+3)$$

$$156 = n(51 - 3n)$$
$$3n^2 - 51n + 156 = 0$$
$$n^2 - 17n + 52 = 0$$
$$n^2 - 13n - 4n + 52 = 0$$
$$(n-4)(n-13) = 0 \Rightarrow n = 4, 13$$

When $n = 4$, $S_4 = \frac{4}{2}[2 \times 24 + (4-1)(-3)]$

$$= 2(48 - 9) = 2 \times 39 = 78$$

When $n = 13$, $S_{13} = \frac{13}{2}[2 \times 24 + (13-1)(-3)]$

$$= \frac{13}{2}[48 + (-36)] = 78$$

Hence, the number of terms $n = 4$ or $n = 13$.

146. Find the 20^{th} term of an AP whose 3^{rd} term is 7 and the seventh term exceeds three times the 3^{rd} term by 2. Also find its n^{th} term (a_n).

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d and nth term be a_n.

We have $a_3 = a + 2d = 7$ (1)

$$a_7 = 3a_3 + 2$$

$$a + 6d = 3 \times 7 + 2 = 23 \quad (2)$$

Solving (1) and (2) we have

$$4d = 16 \Rightarrow d = 4$$
$$a + 8 = 7 \Rightarrow a = -1$$
$$a_{20} = a + 19d = -1 + 19 \times 4 = 75$$
$$a_n = a + (n-1)d$$
$$= -1 + 4n - 4$$
$$= 4n - 5.$$

Hence n^{th} term is $4n - 5$.

147. If 7^{th} term of an AP is $\frac{1}{9}$ and 9^{th} term is $\frac{1}{7}$, find 63^{rd} term.

Ans : [Board Term-2 Delhi 2014]

Let the first term be a, common difference be d and nth term be a_n.

We have $a_7 = \frac{1}{9} \Rightarrow a + 6d = \frac{1}{9}$ (1)

$$a_9 = \frac{1}{7} \Rightarrow a + 8d = \frac{1}{7} \quad (2)$$

Subtracting equation (1) from (2) we get

$$2d = \frac{1}{7} - \frac{1}{9} = \frac{2}{63} \Rightarrow d = \frac{1}{63}$$

Substituting the value of d in (2) we get

$$a + 8 \times \frac{1}{63} = \frac{1}{7}$$
$$a = \frac{1}{7} - \frac{8}{63} = \frac{9-8}{63} = \frac{1}{63}$$

Thus $a_{63} = a + (63-1)d$

$$= \frac{1}{63} + 62 \times \frac{1}{63} = \frac{1+62}{63}$$
$$= \frac{63}{63} = 1$$

Hence, $a_{63} = 1$.

148. The ninth term of an AP is equal to seven times the second term and twelfth term exceeds five times the third term by 2. Find the first term and the common difference.

Ans : [Board SQP 2016]

Let the first term be a, common difference be d and nth term be a_n.

Now $a_9 = 7a_2$

$$a + 8d = 7(a + d)$$
$$a + 8d = 7a + 7d$$
$$-6a + d = 0 \quad (1)$$

and $a_{12} = 5a_3 + 2$

$$a + 11d = 5(a + 2d) + 2$$
$$a + 11d = 5a + 10d + 2$$
$$-4a + d = 2 \quad ...(2)$$

Subtracting (2) from (1), we get

$$-2a = -2$$
$$a = 1$$

Substituting this value of a in equation (1) we get

$$-6 + d = 0$$
$$d = 6$$

Hence first term is 1 and common difference is 6.

149. Determine an AP whose third term is 9 and when fifth term is subtracted from 8^{th} term, we get 6.

Ans : [Board Term-2 2015]

Let the first term be a, common difference be d and

nth term be a_n.

We have $\quad a_3 = 9$

$$a + 2d = 9 \quad ...(1)$$

and $\quad a_8 - a_5 = 6$

$$(a + 7d) - (a + 4d) = 6$$

$$3d = 6$$

$$d = 2$$

Substituting this value of d in (1), we get

$$a + 2(2) = 9$$

$$a = 5$$

So, AP is 5, 7, 9, 11, ...

150. Divide 56 in four parts in AP such that the ratio of the product of their extremes (1^{st} and 4^{rd}) to the product of means (2^{nd} and 3^{rd}) is 5:6.

Ans : [Board Term-2 Foreign 2016]

Let the four numbers be $a - 3d, a - d, a + d, a + 3d$

Now $\quad a - 3d + a - d + a + d + a + 3d = 56$

$$4a = 56 \Rightarrow a = 14$$

Hence numbers are $14 - 3d, 14 - d, 14 + d, 14 + 3d$

Now, according to question, we have

$$\frac{(14 - 3d)(14 + 3d)}{(14 - d)(14 + d)} = \frac{5}{6}$$

$$\frac{196 - 9d^2}{196 - d^2} = \frac{5}{6}$$

$$6(196 - 9d^2) = 5(196 - d^2)$$

$$6 \times 196 - 54d^2 = 5 \times 196 - 5d^2$$

$$(6 - 5) \times 196 = 49d^2$$

$$d^2 = \frac{196}{49} = 4$$

$$d = \pm 2$$

Thus numbers are $\quad a - 3d = 14 - 3 \times 2 = 8$

$$a - d = 14 - 2 = 12$$

$$a + d = 14 + 2 = 16$$

$$a + 3d = 14 + 3 \times 2 = 20$$

Thus required AP is 8, 12, 16, 20.

151.

are a, b and c respectively, Show that $a(q - r) + b(r - p) + c(p - q) = 0$.

Ans : [Board Term-2 Foreign 2016]

Let the first term be A and the common difference be D.

$$a = A + (p - 1)D$$

$$b = A + (q - 1)D$$

$$c = A + (r - 1)D$$

Now $\quad a(q - r) = [A + (p - 1)D][q - r]$

$$b(r - p) = [A + (q - 1)D][r - p]$$

and $\quad c[p - q] = [A + (r - 1)D][p - q]$

$a(q - r) + b(r - p) + c(p - q)$

$$= [A + (p - 1)D][q - r] +$$

$$+ [A + (q - 1)D][r - p] +$$

$$+ [A + (r - 1)D][p - q] +$$

$$= A[p - q + q - p + q - r] +$$

$$+ D(p - 1)(q - r) +$$

$$+ D(q - 1)(r - p) +$$

$$+ D(r - 1)(p - q)$$

$$= A[0] +$$

$$+ D[p(q - r) - (q - r)]$$

$$+ D[q(r - p) - (r - p)]$$

$$+ D[r(p - q) - (p - q)]$$

$$= D[p(q - r) + q(r - p) + r(p - q)] +$$

$$- D[(q - r) + (r - p) + (p - q)]$$

$$= D[pq - pr + qr - qp + rp - rq] + 0$$

$$= D[0] = 0$$

152. The sum of n terms of an AP is $3n^2 + 5n$. Find the AP Hence find its 15^{th} term.

Ans : [Board Term-2 2013, 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Now $\quad S_n = 3n^2 + 5n$

$$S_{n-1} = 3(n - 1)^2 + 5\ n - 1$$

$$= 3(n^2 + 1 - 2n) + 5n - 5$$
$$= 3n^2 + 3 - 6n + 5n - 5$$
$$= 3n^2 - n - 2$$
$$a_n = S_n - S_{n-1}$$
$$= 3n^2 + 5n - (3n^2 - n - 2)$$
$$= 6n + 2$$

Thus AP is 8, 14, 20, ...

Now $a_{15} = a + 14d = 8 + 14(6) = 92$

153. For what value of n, are the n^{th} terms of two APs 63, 65, 67, ... and 3, 10, 17, ... equal?

Ans :

Let a, d and A, D be the 1^{st} term and common difference of the 2 APs respectively.
n is same

For 1st AP, $a = 63,\ d = 2$

For 2nd AP, $A = 3, D = 7$

Since nth term is same,

$$a_n = A_n$$
$$a + (n-1)d = A + (n-1)D$$
$$63 + (n-1)2 = 3 + (n-1)7$$
$$63 + 2n - 2 = 3 + 7n - 7$$
$$61 + 2n = 7n - 4$$
$$65 = 5n \Rightarrow n = 13$$

When n is 13, the n^{th} terms are equal i.e., $a_{13} = A_{13}$

154. In an AP the sum of first n terms is $\frac{3n^2}{2} + \frac{13n}{2}$. Find the 25^{th} term.

Ans : [Board Term-2 SQP 2015]

We have $S_n = \dfrac{3n^2 + 13n}{2}$

$$a_n = S_n - S_{n-1}$$
$$a_{25} = S_{25} - S_{24}$$
$$= \frac{3(25)^2 + 13(25)}{2} - \frac{3(24)^2 + 13(24)}{2}$$
$$= \frac{1}{2}\{3(25^2 - 24^2) + 13(25 - 24)\}$$
$$= \frac{1}{2}(3 \times 49 + 13) = 80$$

155. The sum of first n terms of three arithmetic progressions are S_1, S_2 and S_3 respectively. The first term of each AP is 1 and common differences are 1, 2 and 3 respectively. Prove that $S_1 + S_3 = 2S_2$.

Ans : [Board Term-2 OD 2016]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $S_1 = 1 + 2 + 3 + \ldots.. n$

$S_2 = 1 + 3 + 5 + \ldots..$ up to n terms

$S_3 = 1 + 4 + 7 + \ldots..$ upto n terms

Now $S_n = \dfrac{n(n+1)}{2}$

$$S_2 = \frac{n}{2}[2 + (n-1)2] = \frac{n}{2}[2n] = n^2$$

and $S_3 = \dfrac{n}{2}[2 + (n-1)3] = \dfrac{n(3n-1)}{2}$

Now, $S_1 + S_3 = \dfrac{n(n+1)}{2} + \dfrac{n(3n-1)}{2}$

$$= \frac{n[n + 1 + 3n - 1]}{2} = \frac{n[4n]}{2}$$

$= 2n^2 = 2s_2$ Hence Proved

156. If S_n denotes, the sum of the first n terms of an AP prove that $S_{12} = 3(S_8 - S_4)$.

Ans : [Board Term-2 Delhi 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$S_{12} = 6[2a + 11d] = 12a + 66d$$
$$S_8 = 4[2a + 7d] = 8a + 28d$$
$$S_4 = 2[2a + 3d] = 4a + 6d$$
$$3(S_8 - S_4) = 3[(8a + 28d) - (4a + 6d)]$$
$$= 3[4a + 22d] = 12a + 66d$$

$= 6[2a + 11d] = S_{12}$ Hence Proved

157. The 14^{th} term of an AP is twice its 8^{th} term. If the 6^{th} term is -8, then find the sum of its first 20 terms.

Ans : [Board Term-2 OD 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.
Here, $a_{14} = 2a_8$ and $a_6 = -8$

Now $a + 13d = 2(a + 7d)$

$$a+13d = 2a+14d$$

$$a = -d \quad ...(1)$$

and $a_6 = -8$

$$a+5d = -8 \quad ...(2)$$

Solving (1) and (2), we get

$$a = 2, d = -2$$

Now $S_{20} = \frac{20}{2}[2\times 2+(20-1)(-2)]$

$$= 10[4+19\times(-2)]$$

$$= 10(4-38)$$

$$= 10\times(-34) = -340$$

158. If the ratio of the sums of first n terms of two AP's is $(7n+1):(4n+27)$, find the ratio of their m^{th} terms.

Ans : [Board Term-2 OD 2016]

Let a, and A be the first term and d and D be the common difference of two AP's, then we have

$$\frac{S_n}{S_n'} = \frac{7n+1}{4n+27}$$

$$\frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}[2A+(n-1)D]} = \frac{7n+1}{4n+27}$$

$$\frac{2a+(n-1)d}{2A+(n-1)D} = \frac{7n+1}{4n+27}$$

$$\frac{a+\left(\frac{n-1}{2}\right)d}{A+\left(\frac{n-1}{2}\right)D} = \frac{7n+1}{4n+27}$$

Substituting $\frac{n-1}{2} = m-1$ or $n = 2m-1$ we get

$$\frac{a+(m-1)d}{A+(m-1)D} = \frac{7(2m-1)+1}{4(2m-1)+27} = \frac{14m-6}{8m+23}$$

Hence, $\frac{a_m}{A_m} = \frac{14m-6}{8m+23}$

159. If the sum of the first n terms of an AP is $\frac{1}{2}[3n^2+7n]$, then find its n^{th} term. Hence write its 20^{th} term.

Ans : [Board Term-2 Delhi 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Sum of n term, $S_n = \frac{1}{2}[3n^2+7n]$

Sum of 1 term, $S_1 = \frac{1}{2}[3\times(1)^2+7(1$

$$= \frac{1}{2}[3+7] = \frac{1}{2}\times 10 = 5$$

Sum of 2 term, $S_2 = \frac{1}{2}[3(2)^2+7\times 2]$

$$= \frac{1}{2}[12+14] = \frac{1}{2}\times 26 = 13$$

Now $a_1 = S_1 = 5$

$$a_2 = S_2 - S_1 = 13-5 = 8$$

$$d = a_2 - a_1 = 8-5 = 3$$

Now, AP is $5, 8, 11, ...$

n^{th} term, $a_n = a+(n-1)d$

$$= 5+(n-1)3$$

$$= 5+(20-1)(3)$$

$$= 5+57$$

$$= 62$$

Hence, $a_2 = 62$

160. In an AP, if the 12^{th} term is -13 and the sum of its first four terms is 24, find the sum of its first ten terms.

Ans : [Board Term-2 Foreign 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$a_{12} = a+11d = -13 \quad ...(1)$$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

Now $S_4 = 2[2a+3d] = 24$

$$2a+3d = 12 \quad ...(2)$$

Multiplying (1) by 2 and subtracting (2) from it we get

$$(2a+22d)-(2a+3d) = -26-12$$

$$19d = -38$$

$$d = -2$$

Substituting the value of d in (1) we get

$$a+11\times-2 = -13$$

$$a = -13 + 22$$
$$a = 9$$

Now, $$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$S_{10} = \frac{10}{2}(2 \times 9 + 9 \times -2)$$
$$= 5 \times (18 - 18) = 0$$

Hence, $S_{10} = 0$

161. The tenth term of an AP, is -37 and the sum of its first six terms is -27. Find the sum of its first eight terms.

Ans : [Board Term-2 Foreign 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$a_n = a + (n-1)d$$
$$S_n = \frac{n}{2}[2a + (n- \quad)d$$
$$a + 9d = -37 \qquad ...(1)$$
$$3(2a + 5d) = -27$$
$$2a + 5d = -9 \qquad ...(2)$$

Multiplying (1) by 2 and subtracting (2) from it, we get

$$(2a + 18d) - (2a + 5d) = -74 + 9$$
$$13d = -65$$
$$d = -5$$

Substituting the value of d in (1) we get

$$a + 9 \times -5 = -37$$
$$a = -37 + 45$$
$$a = 8$$

Now $$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$= \frac{8}{2}[2 \times 8 + (8-1)(-5)]$$
$$= 4[16 - 35]$$
$$= 4 \times -19 = -76$$

Hence, $S_n = -76$

162. Find the sum of first seventeen terms of AP whose 4^{th} and 9^{th} terms are -15 and -30 respectively.

Ans : [Board Term-2 2014]

Let the first term be a, common difference be d and nth term be a_n.

Now $$a_4 = a + 3d = -15 \qquad ...(1)$$
$$a_9 = a + 8d = -30 \qquad ...(2)$$

Subtracting eqn (1) from eqn (2), we obtain

$$(a + 8d) - (a + 3d) = -30 - (-15)$$
$$5d = -15 \Rightarrow d = \frac{-15}{5} = -3$$

Substituting the value of d in (1) we get

$$a + 3d = -15$$
$$a + 3(-3) = -15$$
$$a = -15 + 9 = -6$$

Now $$S_{17} = \frac{17}{2}[2 \times (-6) + (17-1)(-3)]$$
$$= \frac{17}{2}[-- 12 + 16 \times (-3)]$$
$$= \frac{17}{2}[-12 - 48]$$
$$= \frac{17}{2}[-60] = 17 \times (-30)$$
$$= -510$$

Thus $S_{17} = -510$

163. The common difference of an AP is -2. Find its sum, if first term is 100 and last term is -10.

Ans : [Board Term-2 2014]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $$a = 100, d = -2, t_n = -10$$

Now $$a_n = a + (n-1)d$$
$$-10 = 100 + (n-1)(-2)$$
$$-10 = 100 - 2n + 2$$
$$2n = 112$$
$$n = 56$$

Thus 56^{th} term is -10 and number of terms in AP are 56.

Now $$S_n = \frac{n}{2}(a + 1)$$
$$S_{56} = \frac{56}{2}(100 - 10)$$

$$= \frac{56}{2}(90) = 56 \times 45 = 2520$$

Thus $S_n = 2520$

164. The 16^{th} term of an AP is five times its third term. If its 10^{th} term is 41, then find the sum of its first fifteen terms.

Ans : [Board Term-2 OD 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have, $a_{16} = 5a_3$

$$a + 15d = 5(a + 2d)$$

$$4a = 5d \quad ...(1)$$

and $a_{10} = 41$

$$a + 9d = 41 \quad ...(2)$$

Solving (1) and (2), we get $a = 5, d = 4$

Now $S_{15} = \frac{15}{2}[2 \times 5 + (15-1) \times 4]$

$$= \frac{15}{2}[10 + 56]$$

$$= \frac{15}{2} \times 66 = 15 \times 33 = 495$$

Thus $S_{15} = 495$

165. The 13^{th} term of an AP is four times its 3^{rd} term. If the fifth term is 16, then find the sum of its first ten terms.

Ans : [Board Term-2 OD 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Here $a_{13} = 4a_3$

$$a + 12d = 4(a + 2d)$$

$$3a = 4d \quad ...(1)$$

and $a_5 = 16$

$$a + 4d = 16 \quad ...(2)$$

Substituting the value of $a = \frac{4}{3}d$ in (2) we have

$$\frac{4}{3}d + 4d = 16$$

$$16d = 48 \Rightarrow d = 3$$

Thus $a = 4$ and $d = 3$

Now $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{10} = \frac{10}{2}[2 \times 4 + (10-1)3]$$

$$= 5[8 + 27] = 5 \times 35 = 175$$

Thus $S_{10} = 175$

166. The n^{th} term of an AP is given by $(-4n + 15)$. Find the sum of first 20 terms of this AP.

Ans : [Board Term-2 2013]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a_n = -4n + 15$

$$a_1 = -4 \times 1 + 15 = 11$$

$$a_2 = -4 \times 2 + 15 = 7$$

$$a_3 = -4 \times 3 + 15 = 3$$

$$d = a_2 - a_1 = 7 - 11 = -4$$

Now, we have $a = 11$, $d = -4$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{20} = \frac{20}{2}[2 \times 11 + (20-1) \times (-4)]$$

$$= 10[22 - 76]$$

$$= 10 \times (-54) = -540$$

Thus $S_{20} = -540$

167. The sum of first 7 terms of an AP is 63 and sum of its next 7 terms is 161. Find 28^{th} term of AP

Ans : [Board Term-2 Foreign 2014]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Now, $S_7 = 63$

$$\frac{7}{2}[2a + 6d] = 63$$

$$2a + 6d = 18 \quad ...(1)$$

Also, sum of next 7 terms,

$$S_{14} = S_{first7} + S_{next7} = 63 + 161$$

$$\frac{14}{2}[2a + 13d] = 224$$

$$2a + 13d = 32 \quad (2)$$

Subtracting equation (1) form (2) we get

$$7d = 14 \Rightarrow d = 2$$

Substituting the value of d in (1) we get

$$a = 3$$

Now $a_n = a + (n-1)d$

$$a_{28} = 3 + 2 \times (27)$$

$$= 57$$

Thus 28^{th} term is 57.

168. The sum of first n terms of an AP is given by $S_n = 3n^2 - 4n$. Determine the AP and the 12^{th} term.

Ans : [Board Term-2 Delhi 2014, 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$S_n = 3n^2 - 4n$$

$$S_1 = 3(1)^2 - 4(1) = -1$$

$$S_2 = 3(2)^2 - 4(2) = 4$$

$$a_1 = S_1 = -1$$

$$a_2 = S_2 - S_1 = 4 - (-1) = 5$$

$$d = a_2 - a_1 = 5 - (-1) = 6$$

Thus AP is $-1, 5, 11, \ldots$

Now $a_{12} = a + 11d$

$$= -1 + 11 \times 6 = 65$$

169. Find the sum of all two digit natural numbers which are divisible by 4.

Ans : [Board Term-2 Delhi Compt. 2017]

First two digit multiple of 4 is 12 and last is 96

So, $a = 12, d = 4$. Let n^{th} term be last term $a_n = 96$

Now $a + (n-1)d = a_n$

$$12 + (n-1)4 = 96$$

$$(n-1)4 = 96 - 12 = 84$$

$$n - 1 = 21$$

$$n = 21 + 1 = 22$$

Now, $S_{22} = \frac{22}{2}[12 + 96]$

$$= 11 \times 108$$

$$= 1188$$

170. Find the sum of the following series.

$5 + (-41) + 9 + (-39) + 13 + (-37) + 17 + \ldots + (-5) + 81 + (-3)$

Ans : [Board Term-2 Foreign 2017]

The given series can be written as sum of two series

$(5 + 9 + 13 + \ldots + 81) +$

$+(-41) + (-39) + (-37) + (-35) \ldots (-5) + (-3)$

For the series $(5 + 9 + 13 \ldots 81)$

$$a = 5,\ d = 4 \text{ and } a_n = 81$$

Now $a_n = a + (n-1)d$

$$81 = 5 + (n-1)4$$

$$81 = 5 + (n-1)4$$

$$(n-1)4 = 76 \Rightarrow n = 20$$

$$S_n = \frac{20}{2}(5 + 81) = 860$$

For series $(-41) + (-39) + (-37) + \ldots + (-5) + (-3)$

$$a_n = -3,\ a = -41 \text{ and } d = 2$$

$$a_n = -41 + (n-1)(2)$$

$$-3 = -41 + 2n - 2 \Rightarrow n = 20$$

Now $S_n = \frac{20}{2}[-41 + -3] = -440$

Sum of the series $= 860 - 440 = 420$

171. Find the sum of n terms of the series

$\left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right) + \ldots\ldots.$

Ans : [Board Term-2 Delhi 2017]

Let sum of n term be S_n

$$s_n = \left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right) + \ldots\ldots. \text{ up to } n\text{term}$$

$$= (4 + 4 + 4 + \ldots.. \text{ up to } n \text{ terms}) + \left(-\frac{1}{n} - \frac{2}{n} - \frac{3}{n} - \ldots\ldots \text{ up to } n \text{ terms}\right)$$

$$= (4 + 4 + 4 + \ldots. \text{ up to } n \text{ terms}) - \frac{1}{n}(1 + 2 + 3 + \ldots.. \text{ up to } n \text{ terms})$$

$$= 4n - \frac{1}{n} \times \frac{n(n+1)}{2}$$

$$= 4n - \frac{n+1}{2} = \frac{7n-1}{2}$$

Hence, sum of n terms $= \frac{7n-1}{2}$

172. Find the number of multiple of 9 lying between 300 and 700.

Ans : [Board Term-2 OD Compt. 2017]

The numbers, multiple of 9 between 300 and 700 are 306, 315, 324, ... 693.

Let the first term be a, common difference be d and nth term be $a_n = 693$

$$a_n = 306 + (n-1)9$$

$$693 = 306 + (n-1)9$$

$$(n-1)9 = 693 - 306 = 387$$

$$n - 1 = \frac{387}{9} = 43$$

$$n = 43 + 1 = 44$$

Hence there are 44 terms.

173. If the sum of the first 14 terms of an AP is 1050 and its first term is 10 find it 20^{th} term.

Ans : [Board Term-2 OD Compt. 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a = 10$, and $S_{14} = 1050$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{14} = \frac{14}{2}[2 \times 10 + (14-1)d]$$

$$1050 = 7[20 + 13d]$$

$$20 + 13d = \frac{1050}{7} = 150$$

$$13d = 130 \Rightarrow d = 10$$

$$a_{20} = a + (n-1)d$$

$$= 10 + 19 \times 10 = 200$$

Hence $a_{20} = 200$

174. If the tenth term of an AP is 52 and the 17^{th} term is 20 more than the 13^{th} term, find AP

Ans : [Board Term-2 OD 2017]

Let the first term be a, common difference be d and nth term be a_n.

Now $a_{10} = 52$

$$a + 9d = 52 \quad ...(1)$$

Also $a_{17} - a_{13} = 20$

$$a + 16d - (a + 12d) = 20$$

$$4d = 20$$

$$d = 5$$

Substituting this valued d in (1), we get

$$a = 7$$

Hence AP is 7, 12, 17, 22, ...

175. Find the sum of all odd number between 0 and 50.

Ans : [Board Term-2 Delhi Compt 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Given AP is $1 + 3 + 5 + 7 + ... + 49$

Let total number of terms be n. Here $a = 1$, $d = 2$ and $a_n = 49$.

$$a_n = 1 + (n-1) \times 2$$

$$49 = 1 + 2n - 2$$

$$50 = 2n \Rightarrow n = 25$$

Now $S_{25} = \frac{n}{2}(a + a_n)$

$$= \frac{25}{2}(1 + 49)$$

$$= 25 \times 25 = 625$$

Hence, Sum of odd number is 625

176. Find the sum of first 15 multiples of 8.

Ans : [Board Term-2 Delhi Compt 2017]

Let the first term be $a = 8$, common difference be $d = 8$, nth term be a_n and sum of n term be S_n.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{15} = \frac{15}{2}[2 \times 8 + (15-1)8$$

$$= \frac{15}{2}[16 + 112]$$

$$= \frac{15}{2} \times 128 = 996$$

Hence, the sum of 15 terms is 960.

177. If m^{th} term of an AP is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$ find the sum of first mn terms.

Ans : [Board Term-2 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Now $$a_m = a+(m-1)d = \frac{1}{n} \quad ...(1)$$

$$a_n = a+(n-1)d = \frac{1}{m} \quad ...(2)$$

Subtracting (2) from (1) we get

$$(m-n)d = \frac{1}{n} - \frac{1}{m} = \frac{m-n}{mn}$$

$$d = \frac{1}{mn}$$

Substituting this value of d in equation (1), we get

$$a = \frac{1}{mn}$$

Now, $$S_{mn} = \frac{mn}{2}\left(\frac{2}{mn} + (mn-1)\frac{1}{mn}\right)$$

$$= 1 + \frac{mn}{2} - \frac{1}{2} = \frac{1}{2} + \frac{mn}{2}$$

$$= \frac{1}{2}[mn+1]$$

Hence, the sum of mn term is $\frac{1}{2}[mn+1]$.

178. How many terms of an AP $9, 17, 25, ...$ must be taken to give a sum of 636?

Ans : [Board Term-2 Delhi Compt 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a = 9, d = 8, S_n = 636$

Now $$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$636 = \frac{n}{2}[18+(n-1)8]$$

$$636 = n[9+(n-1)4]$$

e2

$$636 = n(9+4n-4)$$

$$636 = n(5+4n)$$

$$636 = 5n+4n^2$$

$$4n^2+5n-636 = 0$$

$$4n^2-48n+53n-636 = 0$$

$$4n(n-12)+53(n-12) = 0$$

$$(4n+53)(n-12) = 0$$

Thus $$n = \frac{-53}{4} \text{ or } 12$$

As n is a natural number $n = 12$. Hence 12 terms are required to give sum 636.

179. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.

Ans : [Board Term-2 OD 2014]

The sequence goes like 110, 120, 130, ... 990

Since they have a common difference of 10, they form an AP. Let the first term be a, common difference be d, nth term be a_n.

Here $a = 110,\ a_n = 990,\ d = 10$

$$a_n = a+(n-1)d$$

$$990 = 110+(n-1)\times 10$$

$$990-110 = 10(n-1)$$

$$880 = 10(n-1)$$

$$88 = n-1$$

$$n = 88+1 = 89$$

Hence, there are 89 terms between 101 and 999 divisible by both 2 and 5.

180. How many three digit natural numbers are divisible by 7?

Ans : [Board Term-2 2013]

Let AP is 105, 112, 119, ..., 994 which is divisible by 7.

Let the first term be a, common difference be d, n th term be a_n.

Here, $a = 105, d = 112-105 = 7, a_n = 994$ then

$$a_n = a+(n-1)d$$

$$994 = 105+(n-1)\times 7$$

$$889 = (n-1)\times 7$$

$$n-1 = \frac{889}{7} = 127$$

$$n = 127+1 = 128$$

Hence, there are 128 terms divisible by 7 in AP.

181. How many two digit numbers are divisible by 7?

Ans : [Board Term-2 SQP 2016]

Two digit numbers which are divisible by 7 are 14, 21, 28, ... 98. It forms an AP

Let the first term be a, common difference b
th term be a_n.

Here $a = 14,\ d = 7, a_n = 98$

Now

$$\begin{aligned} a_n &= a+(n-1)d \\ 98 &= 14+(n-1)7 \\ 98-14 &= 7n-7 \\ 84+7 &= 7n \\ 7n &= 91 \Rightarrow n = 13 \end{aligned}$$

182. If the ratio of the 11^{th} term of an AP to its 18^{th} term is 2 : 3, find the ratio of the sum of the first five term of the sum of its first 10 terms.

Ans : [Board Term-2 Delhi Compt. 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Now $$\frac{a_{11}}{a_{18}} = \frac{a+10d}{a+17d} = \frac{2}{3}$$

$$2(a+17d) = 3(a+10d)$$

$$a = 4d \qquad ...(1)$$

Now, $$\frac{S_5}{S_{10}} = \frac{\frac{5}{2}(2a+4d)}{\frac{10}{2}[2a+9d]} = \frac{(a+2d)}{[2a+9d]}$$

Substituting the value $a = 4d$ we have

or, $$\frac{S_5}{S_{10}} = \frac{4d+2d}{8d+9d} = \frac{6}{17}$$

Hence $S_5 : S_{10} = 6:17$

183. How many three digit numbers are such that when divided by 7, leave a remainder 3 in each case?

Ans : [Board Term-2 2012]

When a three digit number divided by 7 and leave 3 as remainder are 101, 108, 115, ... 997

These are in AP. Let the first term be a, common difference be d, nth term be a_n.

Here $a = 101, d = 7, a_n = 997$

Now

$$\begin{aligned} a_n &= a+(n-1)d \\ 997 &= 101+(n-1)7 \\ 997-101 &= 896 = (n-1)7 \\ \frac{896}{7} &= n-1 \\ n &= 128+1 = 129 \end{aligned}$$

Hence, 129 numbers are divided by 7 which leaves remainder is 3.

184. How many multiples of 4 lie between 11 and 266?

Ans : [Board Term-2 2012]

First multiple of 4 is 12 and last multiple of 4 is 264. It forms a AP. Let multiples of 4 be n.

Let the first term be a, common difference be d, n th term be a_n.

Here, $a = 12, a_n = 264, d = 4$

$$\begin{aligned} a_n &= a+(n-1)d \\ 264 &= 12+(n-1)4 \\ n &= \frac{264-12}{4}+1 \end{aligned}$$

Hence, there are 64 multiples of 4 that lie between 11 and 266.

185. Prove that the n^{th} term of an AP can not be n^2+1. Justify your answer.

Ans : [Board Term-2 2015]

Let n^{th} term of AP,

$$a_n = n^2+1$$

Substituting the value of $n = 1, 2, 3, ...$ we get

$$\begin{aligned} a_1 &= 1^2+1 = 2 \\ a_2 &= 2^2+1 = 5 \\ a_3 &= 3^2+1 = 10 \end{aligned}$$

The obtained sequence is 2, 5, 10, 17,......

Its common difference

$$\begin{aligned} a_2-a_1 &= a_3-a_2 = a_4-a_3 \\ 5-2 &\neq 10-5 \neq 17-10 \\ 3 &\neq 5 \neq 7 \end{aligned}$$

Since the sequence has no. common difference, n^2+1 is not a form of n^{th} term of an AP

186. If the p^{th} term of an AP is $\frac{1}{q}$ and q^{th} term is $\frac{1}{p}$. Prove that the sum of first pq term of the AP is $\left[\frac{pq+1}{2}\right]$.

Ans : [Board Term-2 Delhi 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

$$a_p = a+(p-1)d = \frac{1}{q} \qquad ...(1)$$

and $$a_q = a+(q-1)d = \frac{1}{p} \qquad ...(2)$$

Solving (1) and (2) we get

$$a = \frac{1}{pq} \text{ and } d = \frac{1}{p}$$

$$S_{pq} = \frac{pq}{2}\left[2 \times \frac{1}{pq} + (pq-1)\frac{1}{pq}\right] = \frac{pq+1}{2}$$

187. Find the sum of all two digits odd positive numbers.

Ans : [Board Term-2 2014]

The list of 2 digits odd positive numbers are 11, 13 ...... 99. It forms an AP.

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here $a = 11, d = 2, l = 99$

Now
$$a_n = a + (n-1)d$$
$$99 = 11 + (n-1)2$$
$$88 = (n-1)2$$
$$n = 44 + 1 = 45$$
$$S_n = \frac{n}{2}[a + a_n]$$
$$= \frac{45}{2}[11 + 99]$$
$$S_n = \frac{45 \times 110}{2} = 2475$$

Hence the sum of given AP is $S_n = 2475$

188. Find the sum of the two digits numbers divisible by 6.

Ans : [Board Term-2 2013]

Series of two digits numbers divisible by 6 is 12, 18, 24, ...96. It forms an AP. Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here $a = 12, d = 18 - 12 = 6, a_n = 96$

$$a_n = a + (n-1)d$$
$$96 = 12 + (n-1) \times 6$$
$$84 = 6(n-1)$$
$$n = 14 + 1 = 15$$
$$S_n = \frac{n}{2}[a + a_n]$$
$$= \frac{15}{2}[12 + 96]$$
$$= \frac{15 \times 108}{2}$$
$$= 15 \times 54 = 810$$

Hence the sum of given AP is 810.

189. Find the sum of the integers between 100 and 200 that are divisible by 6.

Ans : [Board Term-2 2012]

The series as per question is 102, 108, 114, ... 198. which is an AP.

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here $a = 102, d = 6$ and $l = 198$

Now
$$198 = 102 + (n-1)6$$
$$96 = (n-1)6$$
$$\frac{96}{6} = n - 1$$
$$n = 17$$

Now
$$S_{17} = \frac{n}{2}(a + a_n)$$
$$= \frac{17}{2}[102 + 198]$$
$$= \frac{17}{2} \times 300 = 17 \times 150 = 2550$$

Hence the sum of given AP is 2550.

FOUR MARKS QUESTIONS

190. If the sum of first four terms of an AP is 40 and that of first 14 terms is 280. Find the sum of its first n terms.

Ans : [Board 2019 Delhi]

Let a be the first term and d be the common difference.

Sum of n terms of an AP,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Now $S_4 = 40$ and $S_{14} = 280$

$$\frac{4}{2}[2a + (4-1)d] = 40$$
$$2[2a + 3d] = 40$$
$$2a + 3d = 20 \quad (1)$$

and
$$\frac{14}{2}[2a + (14-1)d] = 280$$
$$7[2a + 13d] = 280$$
$$2a + 13d = 40 \quad (2)$$

Solving equations (1) and (2), we get

$$a = 7 \text{ and } d = 2$$

Now $$S_n = \frac{n}{2}[2 \times 7 + (n-1)2]$$

$$= \frac{n}{2}[14 + 2n - 2]$$

$$= \frac{n}{2}(12 + 2n) = 6n + n^2$$

Hence, sum of n terms is $6n + n^2$.

191. The first term of an AP is 3, the last term is 83 and the sum of all its terms is 903. Find the number of terms and the common difference of the AP.

Ans : [Board 2019 Delhi]

First term, $a = 3$

Last term, $a_n = 83$

Sum of n terms, $S_n = 903$

Since, $$S_n = \frac{n}{2}(a + a_n)$$

$$903 = \frac{n}{2}(3 + 83)$$

$$1806 = 86n$$

$$n = \frac{1806}{86} \Rightarrow n = 21$$

Now $$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$903 = \frac{21}{2}[2 \times 3 + (21-1)d]$$

$$1806 = 21(6 + 20d)$$

$$6 + 20d = 86$$

$$20d = 80 \Rightarrow d = 4$$

Hence, the common difference is 4.

192. Find the common difference of the Arithmetic Progression (AP) $\frac{1}{a}, \frac{3-a}{3a}, \frac{3-2a}{3a}, \ldots (a \neq 0)$

Ans : [Board 2019 OD]

Given AP is $\frac{1}{a}, \frac{3-a}{3a}, \frac{3-2a}{3a}, \ldots \ldots (a \neq 0)$

Here, first term, $a_1 = \frac{1}{a}$

Second term, $a_2 = \frac{3-a}{3a}$

Third term, $a_3 = \frac{3-2a}{3a}$

Common difference,

$$d = a_2 - a_1$$

$$= \frac{3-a}{3a} - \frac{1}{a} = \frac{3-a-3}{3a}$$

$$= \frac{-a}{3a} = \frac{-1}{3}$$

Here, common difference d of given AP is $\frac{-1}{3}$.

193. Which term of the Arithmetic Progression $-7, -12, -17, -22, \ldots$ will be -82? Is -100 any term of the AP ? Given reason for your answer.

Ans : [Board 2019 OD]

Given AP is $-7, -12, -17, -22, \ldots$

Here,

First term, $a_1 = -7$

Second term $a_2 = -12$

Third term, $a_3 = -17$

Common difference,

$$d = a_2 - a_1 = -12 - (-7)$$

$$= -12 + 7 = -5$$

$$d = -5$$

Let a_n be the n^{th} term of AP and it will be -82.

Since, $$a_n = a_1 + (n-1)d$$

$$-82 = -7 + (n-1)(-5)$$

$$-82 = -7 - 5(n-1)$$

$$82 = 5n + 2$$

$$5n = 80 \Rightarrow n = 16$$

Hence, 16^{th} term of AP is -82. Since, these numbers are not factor of 5, hence -100 will not be a term in the given AP.

194. How many terms of the Arithmetic Progression 45, 39, 33, ... must be taken so that their sum is 180? Explain the double answer.

Ans : [Board 2019 OD]

Given AP is 45, 39, 33, ...

Here, $a = 45,\ d = 39 - 45 = -6$ and $S_n = 180$

Now
$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$180 = \frac{n}{2}[2 \times 45 + (n-1)(-6)]$$
$$360 = n(90 - 6n + 6)$$
$$360 = n(96 - 6n)$$
$$60 = n(16 - n)$$
$$n^2 - 16n + 60 = 0$$
$$n^2 - 6n - 10n + 60 = 0$$
$$n(n-6) - 10(n-6) = 0$$
$$(n-10)(n-6) = 0$$
$$n = 10 \text{ or } n = 6$$

Hence, 10 terms or 6 terms can be taken to get the sum of AP as 180.

Now, sum of 6 terms,
$$S_6 = \frac{6}{2}[2 \times 45 + (6-1)(-6)]$$
$$= 3(90 - 30)$$
$$= 3 \times 60 = 180 \quad \text{Hence, verified.}$$

and sum of 10 terms,
$$S_{10} = \frac{10}{2}[2 \times 45 + (10-1)(-6)]$$
$$= 5(90 - 54)$$
$$= 5 \times 36 = 180 \quad \text{Hence, verified.}$$

Here we have two values of n because d is negative. There will be negative terms after some positive terms. Thus first 6 term will give sum 180 and after 10 term it will be again 180 because negative term cancel positive term.

Series will be : 45, 39, 33, 27, 21, 15, 9, 3, -3, -9...

Here it may be easily seen that sum of initial 6 terms is 180. Sum of next 4 terms is zero. Thus sum of 10 terms is also 180.

195. The sum of three numbers in AP is 12 and sum of their cubes is 288. Find the numbers.

Ans : [Board Term-2 Delhi 2016]

Let the three numbers in AP be $a - d,\ a,\ a + d$.
$$a - d + a + a + d = 12$$
$$3a = 12$$
$$a = 4$$

Also, $(4-d)^3 + 4^3 + (4+d)^3 = 288$
$$64 - 48d + 12d^2 - d^3 + 64 + 64 + 48d + 12d^2 + d^3 = 288$$
$$24d^2 + 192 = 288$$
$$d^2 = 4$$
$$d = \pm 2$$

The numbers are 2, 4, 6 or 6, 4,2

196. Find the value of a, b and c such that the numbers $a, 7, b, 23$ and c are in AP

Ans : [Board Term-2 2015]

Let the common difference be d.

Since $a, 7, b, 23$ and c are in AP, we have
$$a + d = 7 \quad ..(1)$$
$$a + 3d = 23 \quad ...(2)$$

Form equation (1) and (2), we get
$$a = -1, d = 8$$
$$b = a + 2d = -1 + 2 \times 8 = -1 + 16 = 15$$
$$c = a + 4d = -1 + 4 \times 8 = -1 + 32 = 31$$

Thus $a = -1, b = 15, c = 31$

197. If S_n denotes the sum of first n terms of an AP, prove that, $S_{30} = 3(S_{20} - S_{10})$

Ans : [Board Term-2 Delhi 2015, Foreign 2014]

Let the first term be a, and common difference be d.

Now
$$S_{30} = \frac{30}{2}(2a + 29d) \quad ...(1)$$
$$= 15(2a + 29d)$$
$$3(S_{20} - S_{10}) = 3[10(2a + 19d) - 5(2a + 9d)]$$
$$= 3[20a + 190d - 10a - 45d]$$
$$= 3[10a + 145d]$$
$$= 15[2a + 29d] \quad ...(2)$$

Hence $S_{30} = 3(S_{20} - S_{10})$

198. The sum of first 20 terms of an AP is 400 and sum of first 40 terms is 1600. Find the sum of its first 10 terms.

Ans : [Board Term-2 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We know $S_n = \frac{n}{2}[2a + (n - \ )d$

Now $S_{20} = \frac{20}{2}(2a + 19d$

$400 = \frac{20}{2}(2a + 19d)$

$400 = 10[2a + 19d]$

$2a + 19d = 40$ (1)

Also, $S_{40} = \frac{40}{2}(2a + 39d)$

$1600 = 20[2a + 39d]$

$2a + 39d = 80$ (2)

Solving equation (1) and (2), we get $a = 1$ and $d = 2$.

Now $S_{10} = \frac{10}{2}[2 \times 1 + (10 - 1)(2)]$

$= 5[2 + 9 \times 2]$

$= 5[2 + 18]$

$= 5 \times 20 = 100$

199. Find $\left(4 - \frac{1}{n}\right) + \left(7 - \frac{2}{n}\right) + \left(10 - \frac{3}{n}\right) + ...$ upto n terms.

Ans : [Board Term-2 2015]

Let sum of n term be S_n, then we have

$s_n = \left(4 - \frac{1}{n}\right) + \left(7 - \frac{2}{n}\right) + \left(40 - \frac{3}{n}\right) + ...$ upto n terms.

$= (4 + 7 + 10 + ... + n \text{ terms}) - \left(\frac{1}{n} + \frac{2}{n} + \frac{3}{n} ..... + 1\right)$

$= (4 + 7 + 10 + ... + n \text{ terms}) - \frac{1}{n}(1 + 2 + 3 + ...n)$

$= \frac{n}{2}[2 \times 4 + (n - 1)(3)] - \frac{1}{n} \times \frac{n}{2}[2 \times 1 + (n - 1)(1)]$

$= \frac{n}{2}[8 + 3n - 3] - \frac{1}{2}[2 + n - 1]$

$= \frac{n}{2}(3n + 5) - \frac{1}{2}(n + 1)$

$= \frac{3n^2 + 5n - n - 1}{2} = \frac{3n^2 + 4n - 1}{2}$

200. Find the 60^{th} term of the AP $8, 10, 12, ...,$ if it has a total of 60 terms and hence find the sum of its last 10 terms.

Ans : [Board Term-2 OD 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

We have $a = 8, d = 10 - 8 = 2$

$a_n = a + (n - 1)d$

Now $a_{60} = 8 + (60 - 1)2 = 8 + 59 \times 2 = 126$

and $a_{51} = 8 + 50 \times 2 = 8 + 100 = 108$

Sum of last 10 terms,

$S_{51-60} = \frac{n}{2}(a_{51} + a_{60})$

$= \frac{10}{2}(108 + 126)$

$= 5 \times 234 = 1170$

Hence sum of last 10 terms is 1170.

201. An arithmetic progression $5, 12, 19, ...$ has 50 terms. Find its last term. Hence find the sum of its last 15 terms.

Ans : [Board Term-2 OD 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a = 5,\ d = 12 - 5 = 7$ and $n = 50$

$a_{50} = 5 + (50 - 1)7$

$= 5 + 49 \times 7 = 348$

Also the first term of the AP of last 15 terms be a_{36}

$a_{36} = 5 + 35 \times 7$

$= 5 + 245 = 250$

Now, sum of last 15 terms,

$S_{36-50} = \frac{15}{2}[a_{36} + a_{50}]$

$= \frac{15}{2}[250 + 348]$

$= \frac{15}{2} \times 598 = 4485$

Hence, sum of last 15 terms is 4485.

202. If the sum of first n term of an an AP is given by

$S_n = 3n^2 + 4n$. Determine the AP and the n^{th} term.

Ans : [Board Term-2 2014]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $S_n = 3n^2 + 4n.$

$$a_1 = 3(1)^2 + 4(1) = 7$$

$$a_1 + a_2 = S_2 = 3(2)^2 + 4(2$$

$$= 12 + 8 = 20$$

$$a_2 = S_2 - S_1 = 20 - 7 = 13$$

$$a + d = 13$$

or, $7 + d = 13$

Thus $d = 13 - 7 = 6$

Hence AP is $7, 13, 19, ...$

Now, $a_n = a + (n-1)d$

$$= 7 + (n-1)(6)$$

$$= 7 + 6n - 6$$

$$= 6n + 1$$

$$a_n = 6n + 1$$

203. The sum of the 3^{rd} and 7^{th} terms of an AP is 6 and their product is 8. Find the sum of first 20 terms of the AP.

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

We have $a_3 + a_7 = 6$

$$a + 2d + a + 6d = 6$$

$$a + 4d = 3 \quad (1)$$

and $a_3 \times a_7 = 8$

$$(a + 2d)(a + 6d) = 8 \quad (2)$$

Substituting the value $a = (3 - 4d)$ in (2) we get

$$(3 - 4d + 2d)(3 - 4d + 6d) = 8$$

$$(3 + 2d)(3 - 2d) = 8$$

$$9 - 4d^2 = 8$$

$$4d^2 = 1 \Rightarrow d^2 = \frac{1}{4} \Rightarrow d = \pm\frac{1}{2}$$

CASE 1 : Substituting $d = \frac{1}{2}$ in equation (1), $a = 1$.

$$S_{20} = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{20}{2}\left[2 + \frac{19}{2}\right] = 115$$

Thus $d = \frac{1}{2}, a = 1$ and $S_{20} = 115$

CASE 2 : Substituting $d = -\frac{1}{2}$ in equation (1) $a = 5$

$$S_{20} = \frac{20}{2}\left[2 \times 5 + 19 \times \left(-\frac{1}{2}\right)\right]$$

$$= 10\left[10 - \frac{19}{2}\right] = 15$$

Thus $d = -\frac{1}{2}, a = 5$ and $S_{20} = 15$

204. If the sum of first m terms of an AP is same as the sum of its first n terms $(m \neq n)$, show that the sum of its first $(m + n)$ terms is zero.

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n, and sum of n term be S_n

Now $S_m = S_n$

$$\frac{m}{2}[2a + (m-1)d] = \frac{n}{2}[2a + (n- \quad)d$$

$$2ma + m(m-1)d = 2na + n(n- \quad)d$$

$$2a(m-n) + [(m^2 - n^2) - m + n]\, d = 0$$

$$2a(m-n) + [(m-n)(m+n) - (m-n)]\, d = 0$$

$$(m-n)[2a + (m+n-1)d] = 0$$

$$2a + (m+n-1)d = 0 \qquad [m - n \neq 0]$$

$$S_{m+n} = \frac{m+n}{2}[2a + (m+n-1)d]$$

$$= \frac{m+n}{2} \times 0 = 0$$

205. If $1 + 4 + 7 + 10 ..... + n = 287$, Find the value of n.

Ans : [Board 2020 Std, Board Term-2 Foreign 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a = 1, d = 3$ and $S_n = 287$.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\frac{n}{2}[2 \times 1 + (n-1)3] = 287$$

$$\frac{n}{2}[2 + (3n - 3)] = 287$$

$$3n^2 - n = 574$$
$$3n^2 - n - 574 = 0$$
$$3n^2 - 42n + 41n - 574 = 0$$
$$3n(n-14) + 41(n-14) = 0$$
$$(n-14)(3n+41) = 0$$

Since negative value is not possible, $n = 14$

$$a_{14} = a + (n-1)d$$
$$= 1 + 13 \times 3 = 40$$

206. Find the sum of first 24 terms of an AP whose n^{th} term is given by $a_n = 3 + 2n$.

Ans : [Board Term-2 OD Comptt. 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

We have $a_n = 3 + 2n$

$$a_1 = 3 + 2 \times 1 = 5$$
$$a_2 = 3 + 2 \times 2 = 7$$
$$a_3 = 3 + 2 \times 3 = 9$$

Thus the series is 5, 7, 9, ... in which

$$a = 5 \text{ and } d = 2$$

Now $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{24} = \frac{24}{2}(2 \times 5 + 23 \times 2)$$
$$= 12 \times 56$$

Hence, $S_{24} = 672$.

207. Find the number of terms of the AP $-12, -9, -6, ..., 21$. If 1 is added to each term of this AP, then find the sum of all the terms of the AP thus obtained.

Ans : [Board Term-2 2013]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

We have $a = -12, d = -9 - (-12) = 3$

$$a_n = a + (n-1)d$$
$$21 = -12 + (n-1) \times 3$$
$$21 + 12 = (n-1) \times 3$$
$$33 = (n-1) \times 3$$
$$n - 1 = 11$$
$$n = 11 + 1 = 12$$

Now, if 1 is added to each term we have a new AP with $-12+1, -a+1, -6+1......21+1$

Now we have $a = -11, d = 3$ and $a_n = 22$ and $n = 12$

Sum of this obtained AP,

$$S_{12} = \frac{12}{2}[-11 + 22]$$
$$= 6 \times 11 = 66$$

Hence the sum of new AP is 66.

208. How many terms of the AP $-6, \frac{11}{2}, -5, ...$ are needed to given the sum -25? Explain the double answer.

Ans : [Board Term-2 2012]

AP is $-6, -\frac{11}{2}, -5 ....$

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here we have $a = -6$

$$d = -\frac{11}{2} + \frac{6}{1} = \frac{1}{2}$$
$$S_n = -25$$
$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$-25 = \frac{n}{2}\left[-12 + (n-1) \times \frac{1}{2}\right]$$
$$-50 = n\left[\frac{-24 + (n-1)}{2}\right]$$
$$-100 = n[n-25]$$
$$n^2 - 25n + 100 = 0$$
$$(n-20)(n-5) = 0$$
$$n = 20, 5$$

or, $S_{20} = S_5$

Here we have got two answers because two value of n sum of AP is same. Since a is negative and d is positive; the sum of the terms from 6^{th} to 20^{th} is zero.

209. If S_1, S_2, S_3 be the sum of $n, 2n, 3n$ terms respectively of an AP, prove that $S_3 = 3(S_2 - S_1)$.

Ans : [Board Term-2 2012]

Let the first term be a, and common difference be d.

Now $S_1 = \frac{n}{2}[2a+(n-1)d]$

$$S_2 = \frac{2n}{2}[2a+(2n-1)d]$$

$$S_3 = \frac{3n}{2}[2a+(3n-1)d]$$

$$3(S_2 - S_1) = 3\left[\frac{2n}{2}[2a+(2n-1)d] - \frac{n}{2}[2a+ n-1 \ d\ ]\right]$$

$$= 3\left[\frac{n}{2}[4a+2(2n-1)d] - [2a+ n-1 \ d\ ]\right]$$

$$= 3\left[\frac{n}{2}(4a+4nd-2d-2a-nd+d\right.$$

$$= 3\left[\frac{n}{2}(2a+3nd-d)\right]$$

$$= \frac{3n}{2}[2a+(3n-1)d] = S_3$$

210. An AP consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the past three terms is 429. Find the AP.

Ans : [Board Term-2 SQP 2017]

Let the middle most terms of the AP be $(x-d)$, x and $(x+d)$.

We have $x-d+x+x+d = 225$

$$3x = 225 \Rightarrow x = 75$$

and the middle term $= \frac{37+1}{2} = 19^{th}$ term

Thus AP is

$(x-18d), \ldots.(x-2d), (x-d), x, (x+d), (x+2d), \ldots\ldots.$

$(x-18d)$

Sum of last three terms,

$$(x+18d)+(x+17d)+(x+16d) = 429$$

$$3x+51d = 429$$

, $$225+51d = 429 \Rightarrow d = 4$$

First term $a_1 = x-18d = 75-18\times 4 = 3$

$$a_2 = 3+4 = 7$$

Hence AP = 3, 7, 11, ……., 147.